MATH 131
Final Exam
December 14, 1987

Name KEY

TMBOK $2\frac{1}{2}$ aft 2h

S-U ? ☐

Weight of exam: Equal one test ☐ Double ☐

slightly short

1. Complete the following DEFinition: (5)

$y = e^x$ if and only if $x = \ln y$

2. Evaluate the following finite or infinite limits. If two-sided limits do not exist, find one-sided limits (if they exist). Show work. (20)

a. $\lim\limits_{x \to +\infty} \dfrac{x^2 - 16}{x + 4} = \lim\limits_{x \to \infty} \dfrac{1 - 16/x^2}{1/x + 4/x^2} = +\infty$

b. $\lim\limits_{x \to -5} \dfrac{x^2 - 25}{x + 5} = \lim\limits_{x \to -5} \dfrac{(x-5)\cancel{(x+5)}}{\cancel{x+5}} = -10$

c. $\lim\limits_{x \to -5^+} \dfrac{x - 5}{x + 5} = -\infty$

-5^+ $\frac{-}{+}$

d. $\lim\limits_{x \to 2^-} f(x)$, where $f(x) = \begin{cases} x^2 + 1, & x \le 2 \\ x + 1, & x \ge 2 \end{cases}$

$= \cancel{4}\,5$

1:40

3. Find the equation of the straight line which is tangent to the curve $ye^x + y^2 = 2$ at the point (0, 1). (9)

$ye^x + e^x y' + 2yy' = 0$

$1 + y' + 2y' = 0$

$3y' = -1$

$y' = -\frac{1}{3}$

$y - 1 = -\frac{1}{3}x$

$y = -\frac{1}{3}x + 1$

2:00

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Find the derivative of each of the following: (20)

a. $f(x) = \dfrac{\tan x}{e^x - x}$

$$f' = \frac{(e^x - x)\sec^2 x - \tan x\,[e^x - 1]}{(e^x - x)^2}$$

most

b. $f(x) = e^{3x^2} + \log_3 x$

$\dfrac{\ln x}{\ln 3}$

$$f' = e^{3x^2} \cdot 6x + \frac{1}{x \ln 3}$$

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c. $f(x) = 4x^2 - x + e^3$

$$f' = 8x - 1 + 0$$

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d. $x^2y^3 - \ln y = 5$, find $\frac{dy}{dx}$.

$$x^2 3y^2 y' + 2xy^3 - \frac{1}{y}y' = 0$$

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$$\left(3x^2y^2 - \frac{1}{y}\right)y' = -2xy^3$$

$$y' = \frac{-2xy^3}{3x^2y^2 - 1/y}$$

3:00

5. We can sell 50,000 boxes of cereal at \$1.50 . For each dime increase in price it is estimated that demand will fall by 5000 boxes. What price will maximize total revenue? (10)

$$R = p\left(50{,}000 - 5000\frac{(p - 150)}{10}\right)$$

$$= p(50000 - 500p + 75000)$$

$$= 125{,}000p - 500p^2$$

$$R' = 125{,}000 - 1000p = 0$$

only a few

$p = 125$ $\qquad p = 150$

1/2 got close

$p = 150$

5:00

not good choice of number

30

6. Compute the following integrals: (36)

a. $\int 3x^3 - \sqrt{x} + 5^x - \cos x \, dx$ $= \int 3x^3 - x^{1/2} + e^{x\ln 5} - \cos x \, dx$

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$= \frac{3x^4}{4} - \frac{x^{3/2}}{3/2} + \frac{e^{x\ln 5}}{\ln 5} - \sin x + C$

$= \frac{3}{4}x^4 - \frac{2}{3}x^{3/2} + \frac{5^x}{\ln 5} - \sin x + C$

b. $\int \frac{3x}{x-3} \, dx$ $= \int \frac{3(u+3)}{u} du = \int 3 + \frac{9}{u} du$

6

$u = x - 3$

$du = dx$

$x = u + 3$

$= 3u + 9\ln|u| + C$

$= 3(x-3) + 9\ln|x-3| + C$

c. $\int \frac{\cos x}{\sqrt{\sin x}} \, dx$ $= \int u^{-1/2} du = 2u^{1/2} + C$

$u = \sin x$

$du = \cos x \, dx$

$= 2\sqrt{\sin x} + C$

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d. $\int \frac{3}{\sqrt{2x-3}} \, dx$ $= \frac{3}{2} \int u^{-1/2} du = \frac{3}{2} \frac{u^{1/2}}{1/2} + C$

$u = 2x - 3$

$du = 2dx$

$\frac{1}{2} du = dx$

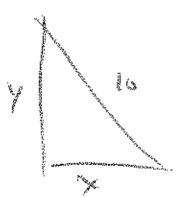
$= 3\sqrt{2x-3} + C$

7:15

7. A 10 foot ladder is leaning against a wall. When the bottom is 5 ft. from the wall the top is sliding down at 3 ft/min. (15)

a. How fast is the bottom moving?

b. Is the area formed by the ladder, wall, and ground increasing or decreasing?



$x^2 + y^2 = 10^2$

$x = 5, \ y = \sqrt{75} = 5\sqrt{3}$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

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$A = xy$

$x = 5$

$10 \frac{dx}{dt} + 10\sqrt{3}(-3) = 0$

$\frac{dA}{dt} = xy' + yx'$

$\frac{dy}{dt} = -3$

$\frac{dx}{dt} = +3\sqrt{3}$

$\boxed{+5.196}$

$= \cancel{5(-3\sqrt{3}) + 5\sqrt{3} \cdot 3}$

$= 5(-3) + 5\sqrt{3} \, 3\sqrt{3}$

I

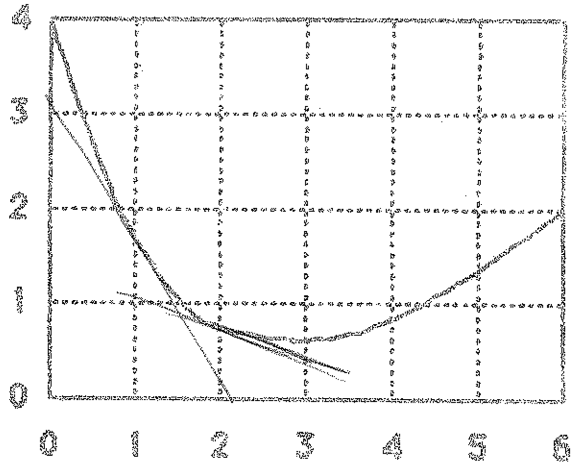
$\overline{51}$

$= -15 + 45$

9:00

$= 30 - 0$

8. The graph of the function f is given below: (20)



a. The range of f is: $[.6, 4]$

b. For what x values is f increasing? decreasing? $x < 3$ D
$x > 3$ I

c. For what x values is the graph of f concave up? down?
all x $[0,6]$

d. What are the absolute maximum and minimum values of f? max 4
min .6

e. $f'(1) =$ $\frac{.2 - 1.7}{1} = -\frac{1.5}{1} = -1.5$

d. If $g(x) = x^2$, and $h(x) = g(f(x))$, then
i) $h(2) =$ $(f(2))^2 = (.8)^2 = .64$
ii) $h'(2) =$

$2(f(2))^{1} f'(2) = 2(.64)(-.3) =$

$f'(2) = \frac{.4 - .7}{1} = -.3$

11:00

9. Suppose the half-life of a radioactive substance is 7.6 days. If we start with 250 mg:
(15) 14

a. How much will be left after 10 days?
b. How long will it be until there is 10 mg left?

$f(x) = f(0)e^{kt}$
$= 250 e^{kt}$
$\frac{1}{2} = e^{k(7.6)}$
$\ln \frac{1}{2} = k(7.6)$
$k = \frac{\ln 1/2}{7.6}$

$f(x) = 250 e^{\frac{\ln(1/2)}{7.6} t}$

a. $f(10) = 250 e^{\frac{\ln(1/2)}{7.6} \cdot 10}$
$\boxed{100.43}$

b. $10 = 250 e^{\frac{\ln(1/2)}{7.6} t}$
$\ln \frac{1}{25} = \frac{\ln 1/2}{7.6} t$
$t = \boxed{35.29}$

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12:20

10. Compute $\log_7 16$. (5)

$$\frac{\ln 16}{\ln 7} = 1.42483$$

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11. Suppose we have $1000 to invest. If we can get an interest rate of 7%: (15)
   a. How much will we have at the end of 2 years if interest is compunded:
      i) monthly?
      ii) continuously?
   b. How long will it be until we have $1200? (Compounding monthly.)

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$A = 1000\left(1+\frac{.07}{12}\right)^{24}$ $\boxed{1149.80}$

$A = 1000\, e^{(.07)(2)}$ $\boxed{1150.27}$

$1000\left(1+\frac{.07}{12}\right)^{12t} = 1200$

$12t \ln\left(1+\frac{.07}{12}\right) = \ln(1.2)$

$t = \frac{\ln(1.2)}{12\ln(\;)}$ $\boxed{2.61}$

13:25

12. Carefully sketch the graphs of the following on the next page. Give coordinates of maxima, minima, points of inflection. Give equations of asymptotes. (30)
   a. $f(x) = \ln(1+x^2)$.
   b. $f(x) = \frac{1+x}{1-x}$

a. $f(x) = \frac{1}{\cancel{x}+x^2}\,2x$

c.p. $x = 0$

$f'(x) = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2}$

$= \frac{2 + 2x^2 - 4x^2}{(\;)^2}$

$= \frac{2 - 2x^2}{(\;)^2}$

P.I. $x = \pm 1$

b. $\frac{(1-x) - (1+x)(-1)}{(1-x)^2}$

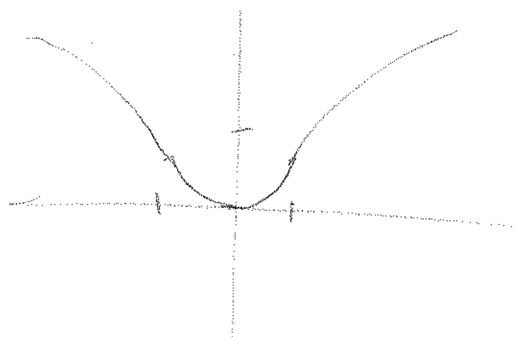
$= \frac{1 - \cancel{x} + 1 + \cancel{x}}{(1-x)^2}$

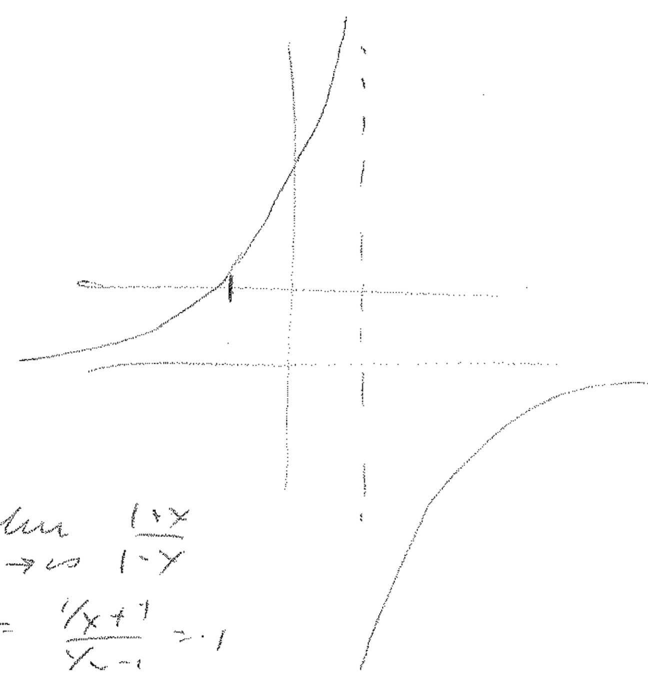
$= \frac{2}{(1-x)^2} = 2(1-x)^{-2}$

$-4(1-x)^{-3}(-1)$

$= \frac{4}{(1-x)^3}$

15:30





$\lim_{x\to\infty} \frac{1+x}{1-x}$

$= \frac{1/x + 1}{1/x - 1} = -1$

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