MATH 131
Quiz II
September 18, 1987

Name KPS

Time OK (or short)
First left aft 10, all by 20
median 15

med = 16

1. Complete the definition: The function f is continuous at the point a

(1) $f(a)$ is defined

(2) $\lim_{x \to a} f(x)$ exists and finite

and (3) $\lim_{x \to a} f(x) = f(a)$.

2. Using limit theorems (and the basic known limits) derive the following limits:

a. $\lim_{x \to 2} \frac{x^2 + 5x}{x - 1}$ $= \frac{\lim_{x\to 2} (x^2+5x)}{\lim_{x\to 2} (x-1)} = \frac{\lim_{x\to 2} x^2 + 5 \lim_{x\to 2} x}{\lim_{x\to 2} x - \lim_{x\to 2} 1}$

$= \frac{2^2+5\cdot 2}{2-1} = \frac{4+10}{1} = \boxed{14}$

b. $\lim_{x \to 0} (2x - 3)(\cos x)$

$= \lim_{x\to 0} (2x-3) \lim_{x\to 0} \cos x$

$= (2(0)-3) \cos 0 = -3 \cdot 1 = \boxed{-3}$

c. $\lim_{x \to \pi/2} \sin(x + \pi/2)$ $\lim_{x\to \pi/2} (x+\pi/2) = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$\lim_{y\to\pi} \sin y = \sin \pi = 0$

so $\lim_{x \to \frac{\pi}{2}} \sin(x+\frac{\pi}{2}) = 0$

3. Evaluate the following finite or infinite limits. If two-sided limits do not exist, find one-sided limits (if they exist).

a. $\lim_{x \to 5} \frac{x^2 - 25}{4(x-5)}$ $= \lim_{x\to 5} \frac{\cancel{(x-5)}(x+5)}{4\cancel{(x-5)}} = \lim_{x\to 5} \frac{(x+5)}{4} = \frac{10}{4} = 2\frac{1}{2}$

b. $\lim_{x \to 2^+} \frac{x + 5}{x - 2}$ $= +\infty$

$\frac{+}{+}$ $x > 2$ so $x-2 > 0$ $x+5>0$ $x-2$ near 0 $x+5$ not near 0

c. $\lim_{x \to 3} f(x)$, where $f(x) = \begin{cases} 2x, & x > 3 \\ x^2 - 2, & x < 3 \end{cases}$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 2x = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x^2-2) = 3^2-2 = 7$

$\lim_{x\to 3} f(x)$ DNE