MATH 131
Test I
September 25, 1987

Name KEY K

ans half left
1 55 min
4 on 5 took 1 hr

1. Using limit theorems (and the basic known limits) derive: (15)

a. $\lim_{x\to\pi/2} \frac{\cos x}{x-1} = \frac{\lim_{x\to\pi/2} \cos x}{\lim_{x\to\frac{\pi}{2}} (x-1)} = \frac{\cos\frac{\pi}{2}}{\frac{\pi}{2}-1} = \boxed{0}$

b. $\lim_{x\to\pi/2} \tan(x+\pi/2) = \boxed{0}$

$\lim_{x\to\pi/2}(x+\pi/2) = \frac{\pi}{2}+\frac{\pi}{2} = \pi$

$\lim_{y\to\pi} \tan y = \tan\pi = 0$

c. $\lim_{x\to 0} \frac{\sin x^2}{x^2}$

$\lim_{x\to 0} x^2 = 0$

$\lim_{y\to 0} \frac{\sin y}{y} = \boxed{1}$

2. Evaluate the following finite or infinite limits. If two-sided limits do not exist, find one-sided limits (if they exist). Show work. (16)

a. $\lim_{x\to-5} \frac{x^2-25}{x+5} = \lim_{x\to-5} \frac{(x-5)\cancel{(x+5)}}{\cancel{x+5}} = \lim_{x\to-5}(x-5) = -5+-5 = -10$

b. $\lim_{x\to-5} \frac{x^2+25}{x+5}$ $\quad \lim_{x\to-5}(x^2+25) = 50$ $\quad$ DNE

$\lim_{x\to-5^+} \frac{x^2+25}{x+5} = +\infty$

$\lim_{x\to-5^-} \frac{x^2+25}{x+5} = -\infty$

c. $\lim_{x\to-2^+} \frac{x+1}{x+2} = -\infty$ $\quad \frac{-2+1}{+}$

d. $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} 2x^2, & x > 2 \\ 2x+4, & x < 2 \end{cases}$ $= 8$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 2x^2 = 8$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(2x+4) = 8$

3. Define tan x and sec x in terms of sine and cosine. (4)

$\tan x = \frac{\sin x}{\cos x}$ $\sec x = \frac{1}{\cos x}$

all but one got

4. Is the following function continuous at the point 1? Explain. (5)

-1

$$f(x) = \begin{cases} 2x+3, & x \geq -1 \\ 2, & x < -1 \end{cases}$$

$f(-1) = 2(-1)+3$
$= 1$ ✓

$\lim_{x \to -1^+} (2x+3) = 1$

$\lim_{x \to -1^-} 2 = 2$

$\neq$ no.

not so good.

5. Find the equations of the vertical aymptotes of the graph of (5)

4

$y = \frac{x}{(x^2-1)(x^2+4)}$, if any.

$x = 1$

$x = -1$

most

6. If $f(x) = \sqrt{x}$, and $g(x) = 2x - 4$, (10)

a. $g(f(4)) = g(\sqrt{4}) = g(2) = 2 \cdot 2 - 4 = 0$

b. $f(f(9)) = f(\sqrt{9}) = f(3) = \sqrt{3}$

c. What is the domain of the function $f \circ g$?

$2x - 4 \geq 0$
$2x \geq 4$
$x \geq 2$

most got

7. Solve for x: $\frac{(x-2)}{x^2(x+5)} \geq 0$ (8)

-5, 0, 2: $+$ | $-$ | $-$ | $+$

~~$x \geq 2$ or $x \leq -5$~~

$x \geq 2$ or $x < -5$

$(-\infty, -5) \cup [2, \infty)$

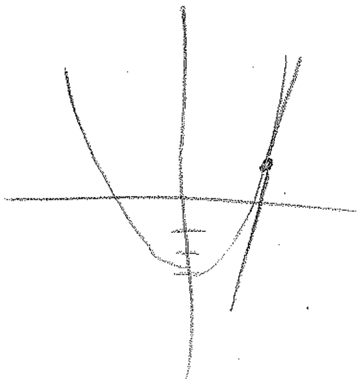
most got

12

8. Let $y = x^2 - 3$. (11)

a. Sketch the curve on the graph paper provided.

b. Find the equation of the line tangent to the curve $y = x^2 - 3$ at the point (2,1), <u>and</u> draw it on the graph in part a.



$$\lim_{x \to 2} \frac{x^2 - 3 - 1}{x - 2} = \lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 4$$

must got.

$$\frac{y-1}{x-2} = 4 \qquad y - 1 = 4(x-2)$$

$$y = 4x - 8 + 1$$

$$\boxed{y = 4x - 7}$$

9. A rock is dropped from a height of 256 feet. Let h(t) denote the distance of the rock from the ground after t seconds. (8)

3 a. The function is $h(t) = 256 - 16t^2$

5 b. What is the velocity of the rock after 1 second?

2/3 got

$$\lim_{t \to 1} \frac{256 - 16t^2 - (256 - 16)}{t - 1}$$

$$= \lim_{t \to 1} \frac{16 - 16t^2}{t - 1} = -16 \lim_{t \to 1} \frac{t^2 - 1}{t - 1}$$

$$= -16 \lim_{t \to 1} \frac{\cancel{(t-1)}(t+1)}{\cancel{t-1}} = -16(2)$$

$$= -32 \text{ ft/sec}$$

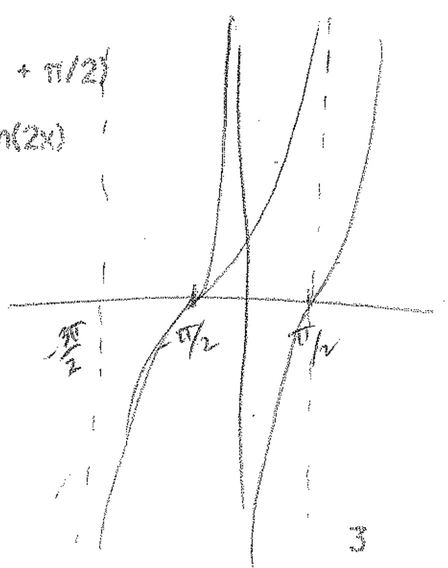
(-1)

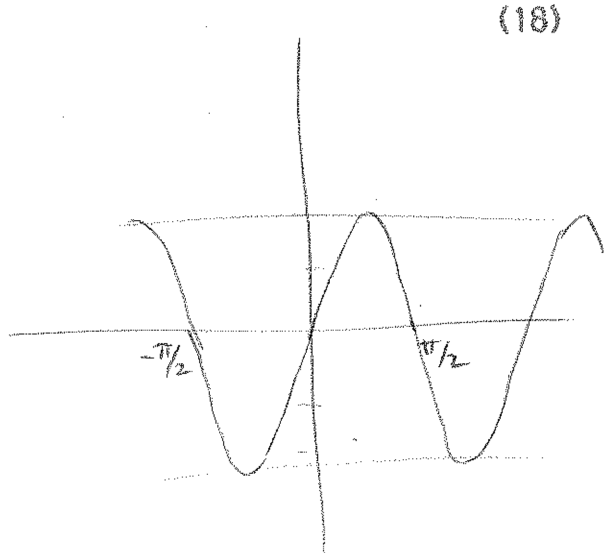
10. Carefully sketch the graph of the following on the graph paper provided. Use straight edge. (18)

60%

a. $y = \tan(x + \pi/2)$

b. $y = 2\sin(2x)$





7