MATH 131
Test II
October 30, 1987

Name Key

Times OK
12-do 1:10

med 79.5
$\bar{x}$ 76.9

1. Find the following derivatives: (18)

a. $f(x) = \dfrac{\cos x}{x^2 - 7}$, $f'(x) =$ $\dfrac{(x^2-7)(-\sin x) - \cos x\,(2x)}{(x^2-7)^2}$

$$= \frac{-\sin x\,(x^2-7) - 2x\cos x}{(x^2-7)^2}$$

b. $g(x) = \sqrt{x^3 - x}$, $g'(x) =$ $\frac{1}{2}(x^3-x)^{-1/2}(3x^2-1)$

$(x^3-x)^{1/2}$

$$= \frac{3x^2-1}{2\sqrt{x^3-x}}$$

almost all got

c. $xy^2 - \sin y = 45$, find $\dfrac{dy}{dx}$.

$$x\,2y\frac{dy}{dx} + y^2 - \cos y\frac{dy}{dx} = 0$$

$$(2xy - \cos y)\frac{dy}{dx} = -y^2 \qquad \frac{dy}{dx} = \frac{-y^2}{2xy - \cos y}$$

all but 3 got

2. Find all critical points of $f(x) = 12x^5 + 15x^4 - 40x^3$. (7)

$$f'(x) = 60x^4 + 60x^3 - 120x^2$$

$$60x^2(x^2 + x - 2) = 0$$

$$x = 0 \qquad (x+2)(x-1)$$

$$\boxed{x = 0, -2, 1} \qquad 0, 1, -2$$

all got

3. Find the equation of the straight line which is tangent to the curve $x^2 + 3x + y^2 - y = -2$ at the point $(-1, 1)$. (10)

$$2x + 3 + 2y\frac{dy}{dx} - \frac{dy}{dx} = 0 \qquad y - 1 = -(x+1)$$

$$-2 + 3 + 2y' - y' = 0 \qquad y = -x - 1 + 1$$

$$y' = -1 \qquad \boxed{y = -x}$$

$$\frac{dy}{dx} = \frac{-(2x+3)}{2y-1}$$

15

1/2

For what ~~x values~~ intervals is each of the following functions increasing? decreasing? (15)

a. $g(x) = 2x^3 - 12x^2 + 18x + 2$

$6x^2 - 24x + 18$

$6(x^2 - 4x + 3)$

8 $6(x-3)(x-1)$

$+ \quad - \quad +$

$1 \qquad 3$

I $x \geq 3$ $\quad x \leq 1$

$[3, \infty)$ $\quad (-\infty, 1]$

D $1 \leq x \leq 3$

$[1, 3]$

$\frac{1}{3}$

b. $h(x) = 2x^3 + 9x^2 + 24x - 3$

7 $6x^2 + 18x + 24$

$6(x^2 + 3x + 4)$

$x = \frac{-3 \pm \sqrt{9-16}}{2}$

no C.P.

I $(-\infty, \infty)$

all x

5. $f(x) = x^3 - 3x^2 - 9x + 3$ has critical points at $x = -1, 3$. Test each as to whether they are relative maxima or minima. Justify (8)

$f' = 3x^2 - 6x - 9$ $\quad f'' = 6x - 6$

$3(x^2 - 2x - 3)$ $\quad x = -1$ $\quad f'' = -12 < 0$ $\quad \cap$ $\quad$ max

$3(x-3)(x+1)$ $\quad x = 3$ $\quad f'' = 12 > 0$ $\quad \cup$ $\quad$ min

6. Consider f below. Find all relative maxima and minima. (10)

$f(x) = x^3 - \frac{x^4}{4}$ $\quad f'(x) = 3x^2 - x^3$ $\quad f''(x) = 6x - 3x^2$

$x^2(3-x)$ $\quad f \quad 3x(2-x)$

C.P. $x = 0$, $x = 3$

$x = 0$ neither

$x = 3$ $\quad f''(3) = 0$ $\quad f'(1) > 0$

$f'(4) < 0$

max

$f''(3) = 18 - \cancel{27} < 0$

$= -9$

33

27

7. Suppose that the cost of producing x units of a product in a day is given by $C = 10 + 4x$ (dollars). If x units are produced, they can be sold at a price of $P = 20 - 2x$ dollars each. (9)

a. What is the marginal cost ?

b. What value of x will maximize total revenue?

c. What value of x will maximize profit?

(a) $\frac{dC}{dx} = 4$

not so good

(b) $R = xP = x(20-2x)$

$= 20x - 2x^2$

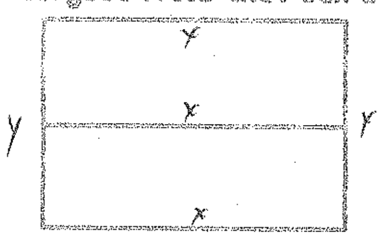
$\frac{dR}{dx} = 20 - 4x$ $\boxed{x = 5}$

(c) $P = 20x - 2x^2 - (10+4x)$

$= -10 + 16x - 2x^2$

$dP/dx = 16 - 4x$ $\boxed{x = 4}$

8. A field is to be fenced as shown below with an additional fence through the middle. The total amount of fencing available is 600 feet. What are the dimensions of the largest field that can be fenced in this way? (10)



$3x + 2y = 600$ $\quad 2y = 600 - 3x$

$A = xy$ $\quad y = 300 - \frac{3}{2}x$

$A = x(300 - \frac{3}{2}x)$ $\quad 0 \le x \le 200$

$= 300x - \frac{3}{2}x^2$

$\frac{dA}{dx} = 300 - 3x$

$x = 100$

$\boxed{100 \times 150}$ $\quad y = 300 - 150$

$= 150$

OR $3x = 600 - 2y$

$x = 200 - \frac{2}{3}y$

$A = y(200 - \frac{2}{3}y) = 200y - \frac{2}{3}y^2$ $\quad 0 \le y \le 300$

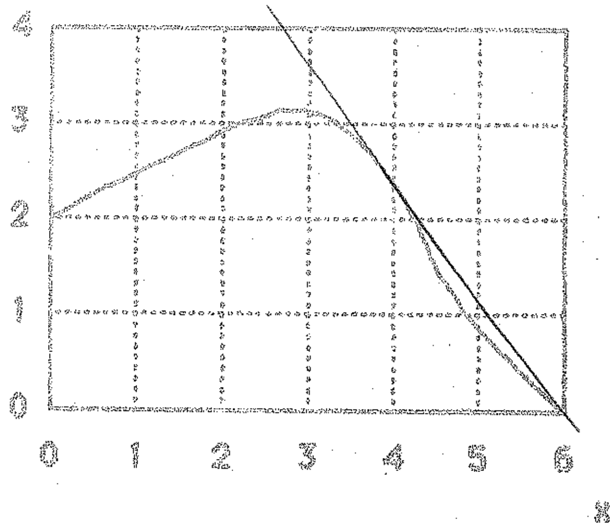
$\frac{dA}{dy} = 200 - \frac{4}{3}y$

$y = 200 \cdot \frac{3}{4} = 150$

$x = 100$

19

9. The function g has the graph below: (13)



a. At approximately what x value does g have its maximum value?
b. What is the minimum value of g?
c. Estimate the derivative of g at x = 4.
d. For what x values is the derivative of g positive?
e. If $f(x) = x^3$, what is $f(g(1))$?
f. If $h(x) = g(x^2)$, find $h'(2)$. ← I got

a. 2.8

b. 0

c. (4, 2.3) (6, 0) $-\frac{2.3}{2} = \boxed{-1.15}$

d. $0 \le x \le 2.8$

e. $g(1) = 2.5$

$f(g(1)) = f(2.5)$
$= (2.5)^3 = \boxed{15.625}$

f. $h'(x) = g'(x^2)\, 2x$

$h'(2) = g'(4)\, 4$
$= (-1.15)(4) = \boxed{4.6}$

13