MATH 131
Test III
December 1, 1997 Name KEY

First left at :40

med = 80 +4 free
$\bar{x} = 74.5$

1. Complete the following DEFinitions (5)

$\ln x = \int_1^x \frac{1}{t}\,dt, \quad x > 0$

2/3

2. Compute the Lower Sum for the function $f(x) = x^2 + 1$ on the interval $[-1,1]$ using the partition $\{-1, 0, .5, 1\}$. (6)

$$LS = [(0)^2+1][1] + (0^2+1)\left(\tfrac{1}{2}\right) + ((.5)^2+1)\tfrac{1}{2}$$
$$= 1 + \frac{1}{2} + \underbrace{\frac{5}{4}\cdot\frac{1}{2}}_{\frac{5}{8}} = \frac{8+4+5}{8} = \boxed{\frac{17}{8} = 2.125}$$

3. Compute the derivative of: (8)

$$f(x) = \sqrt{\ln x} + x e^{2x} = (\ln x)^{1/2} + xe^{2x}$$
$$f'(x) = \frac{1}{2}(\ln x)^{-1/2}\cdot\frac{1}{x} + xe^{2x}\cdot 2 + e^{2x}$$
$$= \frac{1}{2x\sqrt{\ln x}} + 2xe^{2x} + e^{2x}$$
$$\cancel{2x} + e^{2x}(2x+1)$$

3/4

4. Simplify: (5)

$$\ln\sqrt{e^5} = \frac{1}{2}\ln e^5 = \boxed{\frac{5}{2}}$$

1/2

5. Let $f(x) = 3x^5 - 5x^4 + 13$. (9)
a. For what values of x is the graph of f concave up? concave down?
b. Find coordinates of all points of inflection, if any. Justify.

$$f'(x) = 15x^4 - 20x^3$$
$$f''(x) = 60x^3 - 60x^2$$
$$= 60x^2(x-1)$$

1/2

-3
-5
$+13$

(a) CC↑ $x > +1$
CC↓ $x < +1$

(b) $(\cancel{+1}, \cancel{5})$
$(1, 11)$

3
-5
$+13$

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6. If $F(x) = \int_1^x \frac{2}{t^2+1}\,dt$, compute $F'(x)$.

$$\frac{2}{x^2+1}$$

$\frac{1}{4}$

7. Compute the following integrals: (48)

a. $\int x^2 - \sqrt{x} + \sin x + e^x\,dx$ $= \int x^2 - x^{1/2} + \sin x + e^x\,dx$

$\frac{3}{4}$

$$\frac{x^3}{3} - \frac{x^{3/2}}{3/2} - \cos x + e^x + C$$

8 ~~

$$\frac{x^3}{3} - \frac{2}{3}x^{3/2} - \cos x + e^x + C$$

b. $\int_1^4 \sqrt{x} - \frac{3}{\sqrt{x}} + \frac{1}{x}\,dx$ $= \int_1^4 x^{1/2} - 3x^{-1/2} + x^{-1}\,dx$

$\frac{1}{2}$

$$= \frac{x^{3/2}}{3/2} - 3\frac{x^{1/2}}{1/2} + \ln x \Big|_1^4 = \frac{2}{3}4^{3/2} - 6\,4^{1/2} + \ln 4 - \left[\frac{2}{3} - 6 + \ln 1\right]$$

$$= \frac{2}{3}\cdot 8 - 12 + \ln 4 - \frac{2}{3} + 6 - 0$$

$$= \frac{16}{3} - \frac{2}{3} - 12 + 6 + \ln 4 = \frac{14}{3} - 6 + \ln 4 = -\frac{4}{3} + \ln 4$$

.0527

c. $\int \cos(x^2)\,x\,dx$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\int \cos u\,du$$

$$= \frac{1}{2}\sin u + c = \frac{1}{2}\sin x^2 + c$$

$\frac{2}{3}$

d. $\int \sqrt{3x-1}\,dx$ $= \frac{1}{3}\int \sqrt{u}\,du = \frac{1}{3}\int u^{1/2}\,du$

$\frac{2}{3}$

$u = 3x-1$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$= \frac{1}{3}\frac{u^{3/2}}{3/2} + c$$

$$= \frac{2}{9}(3x-1)^{3/2} + c$$

~~= 2~~

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e. $\int_{-2}^{1} \frac{3}{2x+5}\,dx = \int_{x=-2}^{1} \frac{3}{u}\,\frac{1}{2}\,dx = \frac{3}{2}\ln|u| \Big|_{x=-2}^{1}$ $\frac{1}{3}$

$u = 2x+5$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$= \frac{3}{2}\ln|2x+5| \Big|_{-2}^{1}$

$= \frac{3}{2}\ln 7 - \frac{3}{2}\ln 1$

$= \frac{3}{2}\ln 7$

f. $\int 3x\sqrt{x+2}\,dx$ $\frac{1}{2}$

$u = x+2$

$du = dx$

$x = u-2$

$= \int 3(u-2)\sqrt{u}\,du = 3\int u^{3/2} - 2u^{1/2}\,du$

$= 3\left[\frac{u^{5/2}}{5/2} - 2\frac{u^{3/2}}{3/2}\right] + C$

$= \frac{6}{5}(x+2)^{5/2} - 4(x+2)^{3/2} + C$

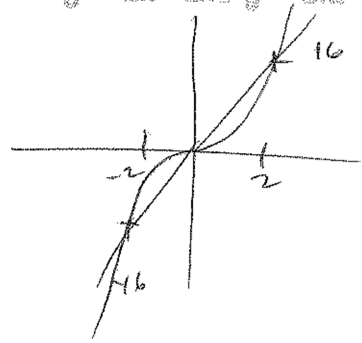
8. Find the area of the region bounded by the curves $y = 2x^3$ and $y = 8x$. (10) $2/3$

$2x^3 = 8x$

$x^3 - 4x = 0$

$x(x^2-4)$

$x = 0, \pm 2$



$$\int_{-2}^{0} 2x^3 - 8x\,dx + \int_{0}^{2} 8x - 2x^3\,dx$$

$$= \frac{2x^4}{4} - \frac{8x^2}{2}\Big|_{-2}^{0} + \frac{8x^2}{2} - \frac{2x^4}{4}\Big|_{0}^{2}$$

$$= -[8-16] + 16 - 8 = 8+8 = 16$$

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