MATH 131
Final Exam
December 13, 1988

Name KEY

time OK
1/2 left by
2 hrs.

S-U ? ☐

Weight of exam: Equal one test ☐ Double ☐

started to leave
at 1:30

all left by 2:20

1. Complete the following DEFinition: (5)

$y = e^x$ if and only if $x = \ln y,\ y > 0$

2. Evaluate the following finite or infinite limits. If two-sided limits do not exist, find one-sided limits (if they exist). Show work. (20)

a. $\lim_{x \to \infty} \frac{x^2 - 16}{x + 4}$ $\lim_{x \to \infty} \frac{1 - \frac{16}{x^2}}{\frac{1}{x} + \frac{4}{x^2}} = +\infty$ or $\lim_{x \to \infty} \frac{x - 16/x}{1 + 4/x} = +\infty$

most

better if not factored?

b. $\lim_{x \to -5} \frac{x^2 - 25}{x + 5}$ $\lim_{x \to -5} \frac{(x-5)\cancel{(x+5)}}{\cancel{x+5}} = -10$

all

c. $\lim_{x \to -5^+} \frac{x - 5}{x + 5}$ $\frac{-5-5}{-5+5} = -\infty$

most

d. $\lim_{x \to 2^-} f(x)$, where $f(x) = \begin{cases} x^2 + 1, & x < 2 \\ x + 1, & x \geq 2 \end{cases}$

almost all

$\lim_{x \to 2^-} (x^2 + 1) = 2^2 + 1 = 5$

3. Find the equation of the straight line which is tangent to the curve $ye^x + y^2 = 2$ at the point (0, 1). (9)

1/2

$ye^x + e^x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$

$1e^0 + e^0 \frac{dy}{dx} + 2 \frac{dy}{dx} = 0$

$3 \frac{dy}{dx} = -1$

$\frac{dy}{dx} = \frac{-1}{3}$

$y - 1 = -\frac{1}{3}(x)$

$\boxed{y = -\frac{1}{3}x + 1}$

34

4. Find the derivative of each of the following: (25)

a. $f(x) = \dfrac{\tan x}{e^x - x}$

$$f'(x) = \frac{(e^x - x)\sec^2 x - \tan x(e^x - 1)}{(e^x - x)^2}$$

$\frac{1}{2}$

b. $f(x) = e^{3x^2} + \log_3 x$

$$= e^{3x^2} + \frac{\ln x}{\ln 3}$$

$$f'(x) = e^{3x^2} \cdot 6x + \frac{1}{x \ln 3}$$

$\frac{1}{2}$

c. $f(x) = 4x^2 - x + e^3$

$$f'(x) = 8x - \cancel{1} + 0$$

$\frac{1}{2}$

d. $x^2y^3 - \ln y = 5$, find $\frac{dy}{dx}$.

$$\left(3x^2y^2 - \frac{1}{y}\right)\frac{dy}{dx} = -2xy^3$$

$\frac{1}{2}$

$$x^2 3y^2 \frac{dy}{dx} + 2xy^3 - \frac{1}{y}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2xy^3}{3x^2y^2 - 1/y}$$

or $$\frac{-2xy^4}{3x^2y^3 - 1}$$

e. $y = \ln(\sin\sqrt{2x})$

$\frac{1}{2}$

$$\frac{dy}{dy} = \frac{1}{\sin\sqrt{2x}} \cos\sqrt{2x} \left(\tfrac{1}{2}(2x)^{-1/2} \cdot 2\right)$$

$$= \frac{\cos\sqrt{2x}}{\sqrt{2x}\,\sin\sqrt{2x}}$$

5. We can sell 110,000 boxes of cereal at $1.50 . For each penny increase in price it is estimated that demand will fall by 500 boxes. What price will maximize total revenue? (10)

x = no of boxes

$$R = \cancel{x}(110{,}000 - 500x)$$
$$= \cancel{110{,}000x - 500x^2}$$

$\frac{1}{2}$

$$R' = 110{,}000 -$$

$$= x(110{,}000 - 500(x - 150))$$
$$= x(110{,}000 - 500x + 75{,}000)$$
$$= x(185{,}000 - 500x)$$
$$= 185{,}000x - 500x^2$$

$$R' = 185{,}000 - 1000x$$
$$(x = 185)$$

x = pennies

$$R = (150 + x)(110{,}000 - 500x)$$
$$R' = (150 + x)(-500) + (110{,}000 - 500x)$$
$$= -75000 - 500x + 110000 - 500x$$
$$0 = 35{,}000 - 1000x$$
$$x = 35 \ \cancel{000}$$
$$\boxed{P = 185}$$

35

6. Compute the following integrals: (36)

a. $\int 3x^3 - \sqrt{x} + 5^x - \cos x \, dx \quad = \int 3x^3 - x^{1/2} + e^{x\ln 5} - \cos x \, dx$

$\checkmark\frac{1}{2}$ $$= \frac{3x^4}{4} - \frac{x^{3/2}}{3/2} + \frac{e^{x\ln 5}}{\ln 5} - \sin x + C$$

$$= \frac{3x^4}{4} - \frac{2}{3}x^{3/2} + \frac{5^x}{\ln 5} - \sin x + C$$

b. $\int \frac{3x}{x-3} \, dx$

$u = x - 3$

$du = dx$

$x = u + 3$

$\frac{1}{2}$

$$= \int \frac{3(u+3)}{u} \, du = \int 3 + \frac{9}{u} \, du$$

$$= 3u + 9\ln|u| + C$$

$$= 3(x-3) + 9\ln|x-3| + C$$

c. $\int \frac{\cos x}{\sqrt{\sin x}} \, dx \quad = \int \frac{1}{\sqrt{u}} \, du = \int u^{-1/2} \, du = \frac{u^{1/2}}{1/2} + C$

$u = \sin x$

$\checkmark\frac{1}{2}$ $du = \cos x \, dx$

$$= 2\sqrt{\sin x} + C$$

d. $\int \frac{3}{\sqrt{2x-3}} \, dx \quad = \frac{1}{2}\int \frac{3}{\sqrt{u}} \, du = \frac{3}{2}\int u^{-1/2} \, du$

$u = 2x - 3$

$du = 2dx$

$$= \frac{3}{2}\,\frac{u^{1/2}}{1/2} + C = 3\sqrt{2x-3} + C$$

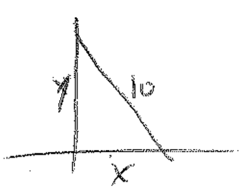
more space

7. A 10 foot ladder is leaning against a wall. When the bottom is 5 ft. from the wall the top is sliding down at 3 ft/min. (15)

$2\frac{1}{2}$

10 a. How fast is the bottom moving?

5 b. Is the area formed by the ladder, wall, and ground increasing or decreasing?



$\frac{dy}{dt} = -3 \qquad x = 5 \qquad y = \sqrt{100-25}$

$= \sqrt{75}$

$= 5\sqrt{3}$

$x^2 + y^2 = 10^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$10\sqrt{3}(-3)$

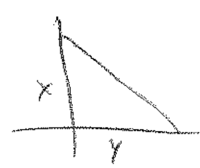
$10(-3) + \cancel{10\sqrt{3}}\;2(-3)\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \cancel{\sqrt{3}\text{ ft/sec}}$

$= \cancel{5\sqrt{3}\text{ ft}}$

$10(5)\frac{dx}{dt} + 2\,5\sqrt{3}(-3) = 0$

$\frac{dx}{dt} = \frac{-(10)3\sqrt{3}}{\cancel{5}(10)} = 3\sqrt{3}$



$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + \frac{1}{2}y\frac{dx}{dt}$

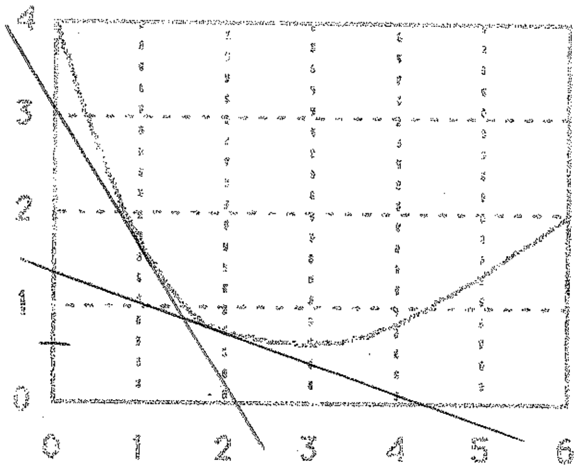
$= \cancel{\frac{1}{2}5\,5\sqrt{3} + \frac{1}{2}3\sqrt{3}(-3)}$

$= \cancel{\frac{1}{2}5\sqrt{3}(5-3) > 0}$ I

$= \frac{1}{2}5(-3) + \frac{1}{2}5\sqrt{3}\,3\sqrt{3}$

$= \frac{15}{2}[-1+3] > 0$ I

8. The graph of the function f is given below: (20)



a. The range of f is: $[.6, 4]$

b. For what x values is f increasing? decreasing? $D[0, 3]$ $I[3, 6]$

c. For what x values is the graph of f concave up? down? all x

d. What are the absolute maximum and minimum values of f? max 4 min .6

e. $f'(1) =$ $f(2) = 0$ $f(0) = 3$ $\frac{0-3}{2-1} = \boxed{-\frac{3}{2}}$

d. If $g(x) = x^2$, and $h(x) = g(f(x))$, then
i) $h(2) =$ $g(f(2)) = (.7)^2 =$ .49
ii) $h'(2) =$
$g'(f(2))\, f'(2)$
$2(.7)(-.32) = -.23$

$f'(2) \approx \frac{0-.7}{4.2-2} = \frac{-.7}{2.2} = -.32$

9. Suppose the half-life of a radioactive substance is 7.6 days. Starting with 250mg:
a. How much will be left after 10 days? (15)
b. How long will it be until there is 10 mg left?

$\frac{1}{2}$

a
$f(t) = 250\, e^{kt}$
$125 = 250\, e^{k7.6}$
$\frac{1}{2} = e^{7.6k}$
$\ln\frac{1}{2} = 7.6k$
$k = \frac{\ln(1/2)}{7.6}$
$f(t) = 250\, e^{\frac{\ln(1/2)}{7.6} t}$

(a) $f(10) = 250\, e^{(\ln 1/2)\frac{10}{7.6}}$
$\boxed{= 100.43 \text{ mg}}$

(b) $10 = 250\, e^{(\ln 1/2)\frac{t}{7.6}}$
$\ln\frac{1}{25} = \ln 1/2 \; t/7.6$
$t = \frac{7.6 \ln(1/25)}{\ln 1/2}$
$= \boxed{35.29}$ days

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10. If the population is 20 million in 1965, and 30 million in 1985, when will it be double? (10)

$f(t) = 20e^{kt}$ $t=0$ 1965

$30 = 20e^{k\,20}$

$\ln\frac{3}{2} = 20k$

$k = \frac{\ln 1.5}{20}$

$f(t) = 20e^{\frac{\ln 1.5}{20}t}$

$40 = 20e^{\frac{\ln 1.5}{20}t}$

$\ln 2 = \frac{\ln 1.5}{20}t$

$t = \frac{20\ln 2}{\ln 1.5} = 34.19$ yrs

1999

$7\frac{1}{3}$

11. Compute $\log_7 16$. (5)

$\frac{\ln 16}{\ln 7} = 1.425$

$\frac{1}{2}$

12. Carefully sketch the graphs of the following on the next page. Give coordinates of maxima, minima, points of inflection. Give equations of asymptotes. (30)

a. $f(x) = \ln(1+x^2)$.

$\frac{1}{3}$

b. $f(x) = \frac{1+x}{1-x}$

$\frac{2}{3}$

$\ln(1+x^2) = 0$

$1+x^2 = 1$

$x^2 = 0$ $x=0$ zero 0

no asym.

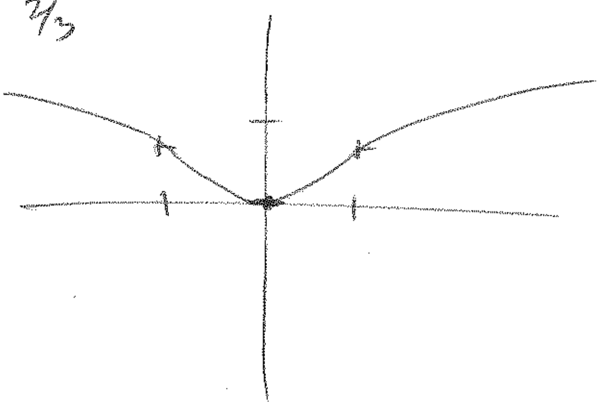
$f'(x) = \frac{1}{1+x^2}2x$ I $x>0$, D $x<0$

$f''(x) = \frac{(1+x^2)2 - 2x(2x)}{(1+x^2)^2} = \frac{2+2x^2-4x^2}{(\ )^2}$

$= \frac{2-2x^2}{(\ )^2}$

$x = \pm 1$

$f(x) = \ln 2 = .69$

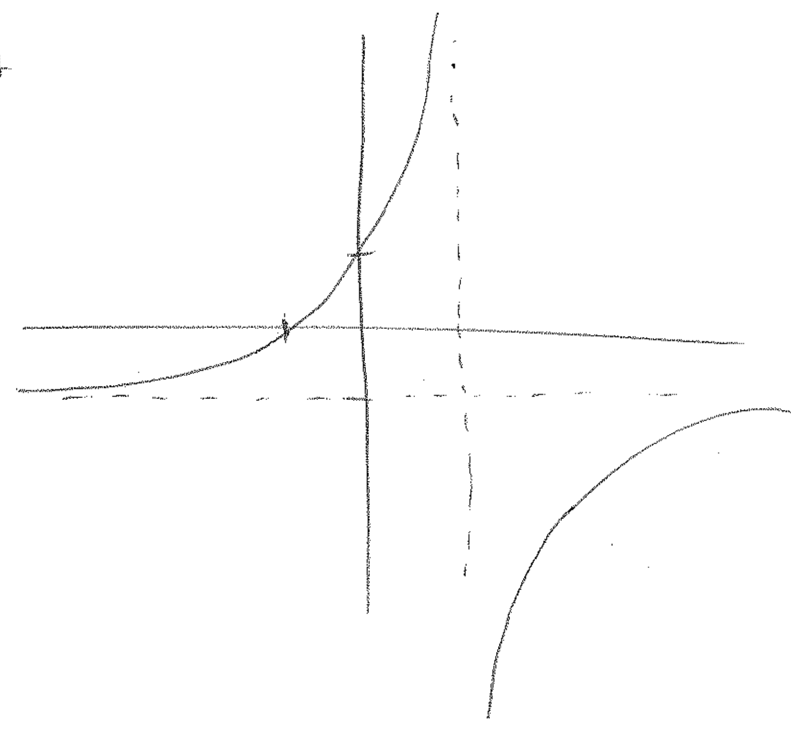


$\lim_{x\to\infty} \frac{1/x+1}{1/x-1} = -1$ $\boxed{y=-1}$ HA

$\boxed{x=1}$ VA

~~no zeros~~

$x=-1$ zero



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