

1. Complete the DEFINITION: The function  $f$  is continuous at the point  $x=a$  means:

$$f(a) \text{ is def}$$

$$\lim_{x \rightarrow a} f(x) \text{ exists}$$

$$f(a) = \lim_{x \rightarrow a} f(x)$$

2 | 999  
 1 | 88  
 1 | 7  
 1 | 66  
 1 | 555  
 1 | 444  
 1 | 33  
 1 | 2  
 1 | 14  
 1 | 100

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2. Use limit theorems to derive the following limits:

a.  $\lim_{x \rightarrow 2} (3x - 5) =$

$$\lim_{x \rightarrow 2} 3x - \lim_{x \rightarrow 2} 5$$

$$= 3 \lim_{x \rightarrow 2} x - 5 = 3(2) - 5 = 1$$

b.  $\lim_{x \rightarrow 2} \frac{2x^2 + 3}{x^2 - x} =$

$$\frac{\lim_{x \rightarrow 2} (2x^2 + 3)}{\lim_{x \rightarrow 2} (x^2 - x)} = \frac{\lim_{x \rightarrow 2} 2x^2 + \lim_{x \rightarrow 2} 3}{\lim_{x \rightarrow 2} x^2 - \lim_{x \rightarrow 2} x}$$

$$= \frac{2 \lim_{x \rightarrow 2} x^2 + 3}{(\lim_{x \rightarrow 2} x)^2 - 2} = \frac{2 \cdot 2^2 + 3}{2^2 - 2} = \frac{11}{2}$$

3. Evaluate the following finite or infinite limits (if two-sided limits do not exist, find one sided limits):

a.  $\lim_{x \rightarrow -5} \frac{x^2 - 5}{x + 5} =$

$$\lim_{x \rightarrow -5^+} \frac{x^2 - 5}{x + 5} = +\infty$$

$$\lim_{x \rightarrow -5^-} \frac{x^2 - 5}{x + 5} = -\infty$$

b.  $\lim_{x \rightarrow 2^+} \frac{x^2 - 5}{x - 2} = -\infty$

c.  $\lim_{x \rightarrow 4} f(x)$ , where  $f(x) = \begin{cases} x^2, & x \geq 4 \\ 3, & x < 4 \end{cases}$

$$\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^+} x^2 = 16$$

$$\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^-} 3 = 3$$

d.  $\lim_{x \rightarrow 3^+} \left[ \frac{1}{x-3} - \frac{1}{x^2-9} \right] =$

$$\lim_{x \rightarrow 3^+} \frac{x+3-1}{(x-3)(x+3)} = \lim_{x \rightarrow 3^+} \frac{x+2}{(x-3)(x+3)} = \frac{5}{0 \cdot 6} = \frac{+}{+} = +\infty$$

4. Find the zeros and equation(s) of vertical asymptotes (if any) for

$f(x) = \frac{2x}{x+2}$ . For what values of  $x$  is  $f$  discontinuous (if any)?

zero  $x=0$  vert asy  $x=-2$  not cont  $x=-2$

5. Is the function in problem 3c continuous at 4?

no limit