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MATH 131
Quiz VII
November 22, 1988 Name KEY

1. Find the following derivative:

$$\frac{d}{dx}\sqrt{\ln x} = \frac{1}{2}(\ln x)^{-1/2}\frac{1}{x}$$

$\frac{1}{2}$ $$= \frac{1}{2x\sqrt{\ln x}}$$

2. Compute the following intergrals:

$$\int_0^1 x^2 + \sin \pi x \, dx = \frac{x^3}{3} - \frac{\cos \pi x}{\pi}\Big|_0^1 = \frac{1}{3} - \frac{\cos \pi}{\pi} - \left(0 - \frac{\cos 0}{\pi}\right)$$

$$= \frac{1}{3} - \left(\frac{-1}{\pi}\right) - \frac{(-1)}{\pi} = \frac{2}{\pi} + \frac{1}{3} =$$

$$\int \frac{\sin x}{\cos^2 x}\, dx = -\int \frac{1}{u^2}\, du = \int -u^{-2}\, du$$

$u = \cos x$

$du = -\sin x \, dx$ $$= -\frac{u^{-\cancel{3}}}{-\cancel{3}} + C = \frac{1}{\cancel{3}\cos^3 x} + C \quad \text{or } \sec x + C$$

$-du = \sin x \, dx$

$$\int \frac{2x - 1}{x^2 - x}\, dx = \int \frac{1}{u}\, du = \ln|u| + c$$

$$= \ln|x^2 - x| + c$$

$u = x^2 - x$

$du = 2x - 1 \, dx$

$$\int \frac{3x}{2x - 3}\, dx = \int \frac{3\left(\frac{u+3}{2}\right)}{u}\, du = \frac{3}{2}\frac{1}{2}\int \frac{u+3}{u}\, du$$

$u = 2x - 3$ $$= \frac{3}{4}\int \left(1 + \frac{3}{u}\right) du$$

$du = 2dx$

$\frac{u+3}{2} = x$ $$= \frac{3}{4}\left(u + 3\ln|u|\right) + c$$

$$= \frac{3}{4}(2x - 3) + \frac{9}{4}\ln|2x - 3| + c$$

3. Find the area of the region between the curves $y = x^2 - 1$ and $y = 3$.

$x^2 - 1 = 3$ $\quad x = 0 \quad y = -1 \quad y = 3$

$x^2 = 4$ $$\int_{-2}^{2} 3 - (x^2 - 1)\, dx = \int_{-2}^{2} 4 - x^2\, dx$$

$x = \pm 2$

$$= 4x - \frac{x^3}{3}\Big|_{-2}^{2} = 8 - \frac{8}{3} - \left(-8 + \frac{8}{3}\right)$$

$$= 16 - \frac{16}{3}$$

$$= \frac{32}{3}$$