med = 79

$\bar{x} = 73.56$ Time just right

MATH 131
Test II
November 4, 1988

Name KEY

1. Find the following derivatives: (16)

Fast left at :46

many checking

a. $f'(x)$ for $f(x) = \cos\sqrt{2x-1}$

$$f'(x) = -\sin\sqrt{2x-1}\ \frac{d}{dx}\sqrt{2x-1}$$
$$= -\sin\sqrt{2x-1}\ \frac{1}{\cancel{2}}(2x-1)^{-1/2}\cdot\cancel{2}$$
$$= -\frac{\sin\sqrt{2x-1}}{\sqrt{2x-1}}$$

$> \frac{1}{2}$

b. $\frac{dy}{dx}$ when $x^2 - 2xy^2 + y^3 = 38$

$$2x - 2\left[x\,2y\frac{dy}{dx} + y^2\right] + 3y^2\frac{dy}{dx} = 0$$

$$\left[3y^2 - 4xy\right]\frac{dy}{dx} = -2x + 2y^2 \qquad \frac{dy}{dx} = \frac{2y^2 - 2x}{3y^2 - 4xy}$$

most

2. Find the following limit: (8)

$$\lim_{x\to\infty}\frac{x(2x+1)}{(x+1)^2} = \lim_{x\to\infty}\frac{2x^2+x}{x^2+2x+1} = \lim_{x\to\infty}\frac{2+1/x}{1+2/x+1/x^2} = 2$$

all but 3

3. Find the <u>equations</u> of the asymptotes (if any) for the graph of (10)

$y = \dfrac{x+5}{(x^2-4)(x^2+1)}$.

vert

$x = 2,\ x = -2$

only a couple went for these

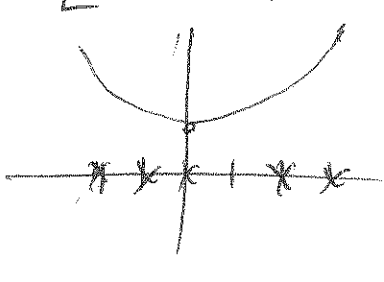
hor

$$\lim_{x\to\infty}\frac{x+5}{(x^2-4)(x^2+1)}$$
$$= \lim_{x\to\infty}\frac{x+5}{x^4-3x^2-4} = \lim_{x\to\infty}\frac{1/x^3+5/x^4}{1-3/x^2-4/x^4}$$
$$= \frac{0}{1} = 0 \qquad \boxed{y=0}$$

all but 5

4. <u>Write out</u> the upper and lower sums for the function $f(x) = x^2 + 1$ for the partition $\{-2, -1, 0, 2, 3\}$ of $[-2,3]$ (8)

$$L = f(-1)\cdot 1 + f(0)\cdot 1 + f(0)\cdot 2 + f(2)\cdot 1$$
$$= ((-1)^2+1)\cdot 1 + 1\cdot 1 + 1\cdot 2 + (2^2+1)\cdot 1$$
$$= 2 + 1 + 2 + 5 = \boxed{10}$$



$$U = f(-2)\cdot 1 + f(-1)\cdot 1 + f(1)\cdot 2 + f(3)\cdot 1$$
$$= ((-2)^2+1)\cdot 1 + ((-1)^2+1)\cdot 1 + (2^2+1)\,2 + (3^2+1)\cdot 1$$
$$= 5 + 2 + 10 + 10 = \boxed{27}$$

most missed

10 × 5

5. Find the slope of the line tangent to the curve $x^2 + 2y^2 = 3$ at the point $(1,1)$. (7)

$$2x + 4y\frac{dy}{dx} = 0$$

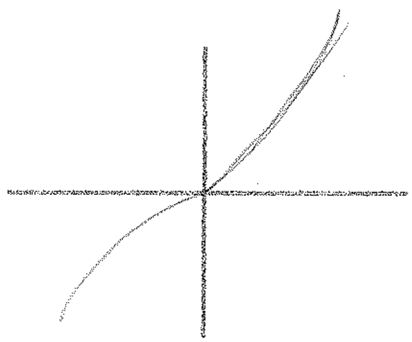
$\frac{1}{2}$

$$2 + 4\frac{dy}{dx} = 0$$

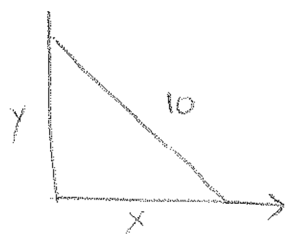
$$\frac{dy}{dx} = -\frac{1}{2}$$

6. Draw the graph of a function that has $f'(x) > 0$ for all x and $f''(x) > 0$ for $x > 0$ and $f''(x) < 0$ for $x < 0$. (6)

$\frac{1}{2}$



7. Suppose a 10 foot ladder is leaning against a wall, and the bottom is being pulled out at the rate of 2 ft./min. How fast is the top moving when the bottom of the ladder is 6 feet from the wall? (10)



$$x^2 + y^2 = 100 \qquad \frac{dx}{dt} = 2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$x = 6$

$y = \sqrt{100-36} = \sqrt{64}$

$= 8$

$(6, 8)$

$\frac{1}{2}$

$$2\cdot 6\,(2) + 2(8)\frac{dy}{dt} = 0$$

$$24 + 16\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = -\frac{24}{16} = \boxed{-\frac{3}{2}}$$

8. Use the tangent line approximation (differentials) to estimate 1/1.97. (7)

$$f(x) = \frac{1}{x}$$

$$f'(x) = -\frac{1}{x^2}$$

$$dy = -\frac{1}{x^2}dx$$

$\frac{1}{3}$

$$dx = -.03$$

$$x = 2$$

$$dy = -\frac{1}{4}(-.03) = +\frac{.03}{4} = .0075$$

$$\frac{1}{1.197} = \frac{1}{2} + .0075 = \boxed{.5075}$$

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9. Find the absolute maximum and minimum values for the function $f(x)=(x-2)^3$ in the interval $0 \le x \le 3$. (8)

$f'(x) = 3(x-2)^2 = 0$

C.P. $x = 2$

EP $x = 0, 3$

x	$f(x)$
0	-8
2	0
3	1

all but 4

max 1 min -8

10. For the following function: (20)

5 a. Find equations of all asymptotes (if any).

9 b. Carefully sketch the graph.

6 c. Find coordinates of zeros, relative (3) extrema, and points (3) of inflection (if any.)

$f(x) = \dfrac{1+x^2}{2+x^2}$

$$f'(x) = \frac{(2+x^2)2x - (1+x^2)(2x)}{(2+x^2)^2}$$

$$= \frac{2x\left[2+x^2-(1+x^2)\right]}{(2+x^2)^2}$$

$$= \frac{2x}{(2+x^2)^2} \quad \text{C.P. } x = 0$$

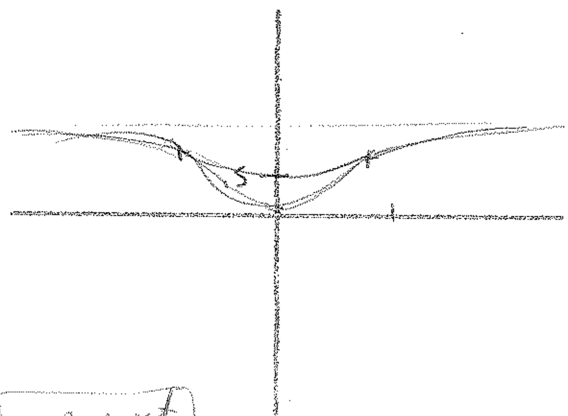
$$f''(x) = \frac{(2+x^2)^2 2 - 2x\,2(2+x^2)2x}{(2+x^2)^2}$$

$$= \frac{2(2+x^2)\left[\overset{2-3x^2}{2+x^2-4x^2}\right]}{(2+x^2)^2}$$

$2-3x^2=0$

$x = \pm\sqrt{\frac{2}{3}}$

$$f(x) = \frac{1+2/3}{2+2/3} = \frac{5/3}{8/3} = 5/8$$



(a) no vert

$$\lim_{x\to\infty} \frac{1+x^2}{2+x^2} = \lim_{x\to\infty}\frac{1/x^2+1}{2/x^2+1} = 1$$

$\boxed{y=1}$

denominator too hard.

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