MATH 131 MOSES

Test IV

November 21, 1988

mean ~~74~~ 73.6

med 76

Name KEY

n1 15

n2 30

Short ?

~~1 long puls short~~

maybe

left shortly at 40

made test 52 min

1/3 45 min

10 < 50 min

10/25

Show work!

1. Complete the defintion: The natural logarithm is defined by (5)

$\frac{1}{2}$

$$\ln x = \int_1^x \frac{1}{t}\,dt\ ,\ x>0$$

2. Write out the upper and lower sums for the function $f(x) = 2x^2 - 5$ for the partition $\{-3, -1, 0, 2, 3\}$ of $[-3,3]$ (8)

$$LS\quad f(-1)\,2 + f(0)\cdot 1 + f(0)\cdot 2 + f(2)\cdot 1$$

$$(2(-1)^2-5)\,2 + (-5)\cdot 1 + (-5)\cdot 2 + (2(2^2)-5)\cdot 1 = -6-5-10+3 = -18$$

$2\frac{1}{2}$

$$US = f(-3)\cdot 2 + f(-1)\cdot 1 + f(2)\cdot 2 + f(3)\cdot 3$$

$$= (2(-3)^2-5)\,2 + (2(-1)^2-5)\cdot 1 + (2(2)^2-5)\cdot 2 + (2(3)^2-5)\cdot 1$$

$$13\cdot 2 + (-3)\cdot 1 + 3\cdot 2 + 13\cdot 1 = 42$$

3. Write out the Riemann sum for the function $f(x) = \ln x$ for the partition $\{1, 2, 4\}$ of $[1,4]$ using midpoints in each subinterval. (6)

$$\ln(1.5)\cdot 1 + (\ln 3)\,2$$

2.238

2.6177

$\frac{7}{2}$

4. Find the following derivatives: (18)

$$\frac{d}{dx}\frac{1}{2x-1} = \quad -(2x-1)^{-2}(2)$$

$$(2x-1)^{-1} \qquad = \frac{-2}{(2x-1)^2}$$

$\frac{1.5}{2.5}$

$$\frac{d}{dx}\sqrt{\ln x} = \frac{d}{dx}(\ln x)^{1/2}$$

$$= \frac{1}{2}(\ln x)^{-1/2}\cdot\frac{1}{x} = \frac{1}{2x\sqrt{\ln x}}$$

R 8:30

$$\frac{d}{dx}\ln(x^3) = \frac{1}{x^3}\,3x^2 = \frac{3}{x}$$

$\frac{1}{2}$

$$\text{or}\quad \frac{d}{dx}\,3\ln x = \frac{3}{x}$$

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5. Compute the following intergrals: (35)

$$\int_0^1 x^2 + \sin \pi x \, dx = \frac{x^3}{3} - \frac{\cos \pi x}{\pi}\Big|_0^1$$

$$= \frac{1}{3} - \frac{\cos \pi}{\pi} - \left(0 - \frac{\cos 0}{\pi}\right)$$

$$= \frac{1}{3} + \frac{1}{\pi} + \frac{1}{\pi} = \frac{1}{3} + \frac{2}{\pi}$$

$$\int \frac{2x-1}{x^2-x} \, dx =$$

must

$$\int \frac{1}{u} \, du = \ln|u| + C$$

$$= \ln|x^2 - x| + C$$

$u = x^2 - x$

$du = 2x - 1 \, dx$

$$\int \frac{2}{3x-4} \, dx = \frac{2}{3}\int \frac{1}{u} \, du = \frac{2}{3}\ln|u| + C$$

$$= \frac{2}{3}\ln|3x-4| + C$$

$u = 3x - 4$

$du = 3dx$

$\frac{1}{3} du = dx$

$$\int \frac{3x}{2x-3} \, dx = \frac{3}{2}\int \frac{3x}{u} \, du = \int \frac{3}{4}\,\frac{u+3}{u} \, du$$

$$= \frac{3}{4}\int \left(1 + \frac{3}{u}\right) du \quad \leftarrow \text{must } (\tfrac{1}{2}) \text{ get to here}$$

$$= \frac{3}{4}\left(u + 3\ln|u|\right) + C$$

$$= \frac{3}{4}(2x-3) + \frac{9}{4}\ln|2x-3| + C$$

$u = 2x - 3$

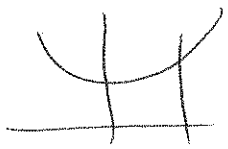
$du = 2dx$

$\frac{1}{2} du = dx$

$2x = u + 3$

$x = \frac{u+3}{2}$

6. Find the area of the region between the curve $y = x^2 + 1$ and the x-axis for x in the interval [0,2]. (6)

must 

$$\int_0^2 x^2 + 1 \, dx = \frac{x^3}{3} + x\Big|_0^2$$

$$= \frac{8}{3} + 2 = \frac{14}{3}$$

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r. For the function $f(x) = (\ln x)^2$: (20)

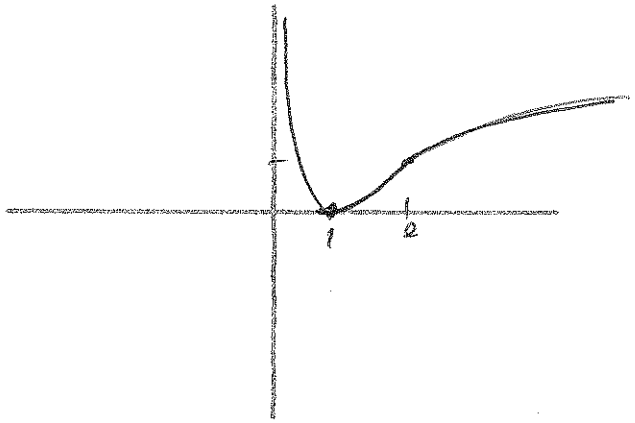
2 a. Why is the domain of f equal to $(0,\infty)$?

8 b. Find critical points (if any).

c. For what x values is the function increasing? decreasing?

4 d. Find coordinates of points of inflection (if any).

6 e. Carefully sketch the graph of f below.



(a) $\ln x > 0$ domain of $\ln x$ is $x > 0$

$x > 1$

(b) $f'(x) = 2(\ln x)\frac{1}{x} = 0$

$\ln x = 0$

$\boxed{x = 1}$ $(1, 0)$

(c) $\ln x > 0$ $\boxed{x > 1}$ ↗

$\ln x < 0$ $\boxed{x < 1}$ ↘

(d) $f''(x) = \dfrac{x\frac{2}{x} - 2\ln x}{x^2} = \dfrac{2 - 2\ln x}{x^2}$

$= \dfrac{2(1 - \ln x)}{x^2}$ $x = e$

$(e, 1)$

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