Time about right.

MATH 131
Test I
September 30, 1988

Name KEY

1. Find the equations of the vertical asymptotes (if any) for the graph of (5)

$y = \frac{x+5}{(x^2-4)(x^2+1)}$. $x^2-4=0$ $x=2$ $x=-2$

$\bar{x} = 76.5$
med = 78.5

$\frac{1}{2}$

3. If $f(x) = \sqrt{x-1}$, and $g(x) = 2x+6$, find: (7)

a. $g(f(2)) =$ 2 $f(2) = \sqrt{1} = 1$ $g(1) = 8$

b. $f(f(26)) =$ 2 $f(26) = \sqrt{25} = 5$ $f(5) = \sqrt{4} = 2$

$\frac{1}{2}$

c. domain of $f \circ g$ 3 $(2x+6)-1 \geq 0$ $2x+5 \geq 0$ $2x \geq -5$ $x \geq -5/2$ $[-5/2, \infty)$

4. Is the following function continuous at $x = -3$? Justify. (5)

$f(x) = \begin{cases} 3x+4, & x \geq -3 \\ 6, & x < -3 \end{cases}$

$\frac{1}{2}$

$f(-3) = 3(-3)+4 = -5$

$\lim_{x \to -3^+} = -5$

$\lim_{x \to -3^-} = 6$ no limit!

5. Find the following limits (possibly $\pm\infty$), or say they do not exist. (18)

a. $\lim_{x \to 5} \frac{2x^2-50}{x-5} = \lim_{x \to 5} \frac{2(x^2-25)}{x-5} = \lim_{x \to 5} \frac{2(x-5)(x+5)}{x-5}$ 10 min

6 each

$= \lim_{x \to 5} 2(x+5) = 2(5+5) = 20$ almost all got this one.

b. $\lim_{x \to -3^+} \frac{x+2}{x+3} =$ $-\infty$ most got

$\frac{-3+2}{+} = \frac{-}{+} = -$

First left after 39 min
The at 50 min
Some done / pass after 5 min
Lots of checking

c. $\lim_{x \to 3} f(x)$, where $f(x) = \begin{cases} x^2, & x > 3 \\ 2x+3, & x < 3 \\ 3, & x = 3 \end{cases}$

$\lim_{x \to 3^+} x^2 = 9$ 9   many got well!

$\lim_{x \to 3^-} (2x+3) = 9$

$246 \bar{x}$
$= \frac{2??}{35}$ 76%

15

6. Let $y = (x + 3)^2$. (10)

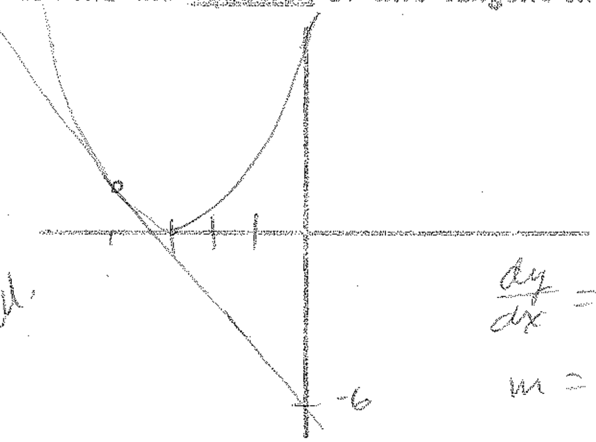
3 a. Sketch the graph.

2 b. Draw the tangent line at the point $x = -4$. ← separate here not line ?

5 c. Find the <u>equation</u> of this tangent line.

lots of trouble here

only few got it all.



$\frac{dy}{dx} = 2(x+3)^1$

$m = 2(-4+3) = -2$

seems to be skipped c but & came back to

~~$y = -2(x+3)$~~

~~$y = -2x-6$~~

$x = -4,\ y = 1$

$y - 1 = -2(x+4)$

$y - 1 = -2x - 8$

$\boxed{y = -2x - 7}$

20

7. Find the derivative of each of the following: (30)

a. $f(x) = 3x^4 + 7\sqrt{x} - \sin x$ $= 3x^4 + 7x^{1/2} - \sin x$

almost all

$f'(x) = 12x^3 + \frac{7}{2}x^{-1/2} - \cos x$

$= 12x^3 + \frac{7}{2\sqrt{x}} - \cos x$

b. $y = (x^3 - x + 1)(x + \cos x)$

5 missed this

$\frac{dy}{dx} = (x^3 - x + 1)(1 - \sin x) + (x + \cos x)(3x^2 - 1)$

c. $f(x) = \frac{x - 2}{x^2 - 4}$

$f'(x) = \frac{(x^2-4)(1) - (x-2)(2x)}{(x^2-4)^2}$

all but 4 got

↑ factor! darn (3 or 4 forgot this one)

d. $y = \sqrt{2x - 3}$ $= (2x-3)^{1/2}$

all but 5 got

$\frac{dy}{dx} = \frac{1}{2}(2x-3)^{-1/2} \cdot 2 = \frac{1}{\sqrt{2x-3}}$

e. Find $f''(x)$ for $f(x) = x^5 + \sin x$.

2 missed

$f'(x) = 5x^4 + \cos x$

$\boxed{f''(x) = 20x^3 - \sin x}$

$\frac{33.3}{40} = \bar{x}$ 83%

30

2. Define tan x in terms of sine and cosine, and find the derivative of tan x from this. (7)

2 $y = \tan x = \frac{\sin x}{\cos x}$ ← must

5 $\frac{dy}{dx} = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$ ↖ many

9. Using only the definition find f'(a) [or f'(x)] for $f(x) = \sqrt{2x-1}$. (6)

$$f'(a) = \lim_{t\to a} \frac{f(t)-f(a)}{t-a} = \lim_{t\to a} \frac{\sqrt{2t-1} - \sqrt{2a-1}}{t-a} \quad \sqrt{}+\sqrt{}\ \ \sqrt{}+\sqrt{}$$

6 get all $= \lim_{t\to a} \frac{(2t-1)-(2a-1)}{(t-a)(\sqrt{2t-1}+\sqrt{2a-1})} = \lim_{t\to a} \frac{2(t-a)}{(t-a)(\sqrt{2t-1}+\sqrt{2a-1})}$

$$= \lim_{t\to a} \frac{2}{\sqrt{2t-1}+\sqrt{2a-1}} = \frac{2}{2\sqrt{2a-1}} = \frac{1}{\sqrt{2a-1}}$$

10. Find the derivative of (10)

1/3 $y = \left(5x^2 + \frac{2}{x}\right)\left(4\sqrt{x} - x\right)\sqrt{2x+5}$. $= (5x^2+2x^{-1})(4x^{1/2}-x)(2x+5)^{1/2}$

$$\frac{dy}{dx} = \left[(5x^2+2x^{-1})(4x^{1/2}-x)\right]\frac{1}{2}(2x+5)^{-1/2}\cdot 2$$

$$+ (2x+5)^{1/2}\left[(5x^2+2x^{-1})(2x^{-1/2}-1) + (4x^{1/2}-x)(10x-2x^{-2})\right]$$

or2 $$(5x^2+2x^{-1})\left[(4x^{1/2}-x)\tfrac{1}{2}(2x+5)^{-1/2}\cdot 2 + (2x+5)^{1/2}(2x^{-1/2}-1)\right]$$

$$+ (4x^{1/2}-x)(2x+5)^{1/2}(10x-2x^{-2})\Big\}$$

14.9 / 25 59%