MATH 131
Final Exam
December 11, 1989

170 total

mean 117/170

Time just ~~fine~~ right
started to leave at 1:24
Most of them left at 2:00

Name Key

Final Exam Double Weight? ______

$$\sum_{i=1}^{n} i = \frac{n\,(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n\,(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \left[\frac{n\,(n+1)}{2}\right]^2$$

1. Find the derivative of: (10)
   a. $x^2 \cos x + \sin x$

   $x^2(-\sin x) + 2x\cos x + \cos x$ — about all

   b. $\dfrac{x^2 - x + 2}{\tan x}$

   $$\frac{\tan x\,(2x-1) - (x^2-x+2)\sec^2 x}{\tan^2 x}$$

2. Write in summation notation: $4 + 9 + 16 + 25 + \ldots + 100$. (5)

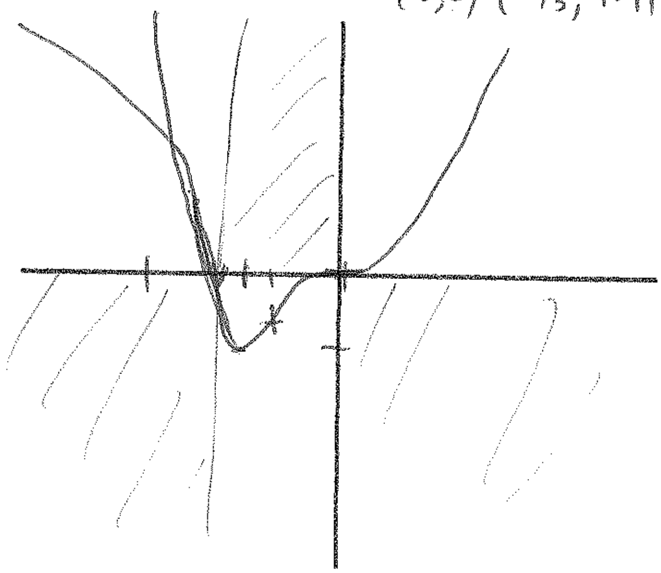
   $\sum_{i=2}^{10} i^2$ — about all

3. What is (5)

   $$\sum_{i=0}^{2} (2i + 3) = 3 + (5) + (7) = 15$$

   many started at 1

4. Sketch the graph of $f(x) = 3x^4 + 4x^3$, and find (if any): (15)
   a. critcal points $(0,0)\ (-1,-1)$
   b. points of inflection $(0,0)\ (-2/3, -.59)$



$f'(x) = 12x^3 + 12x^2 = 0$ — $\frac{1}{2}$

$12x^2(x+1)$

$x = 0, -1$

$f''(x) = 36x^2 + 24x$

$12x(3x + 2)$

$x = 0,\ x = -\frac{2}{3}$

zeros $x^3(3x+4) = 0$

$x = 0,\ x = -\frac{4}{3}$

$+ \quad -4/3 \quad - \quad 0 \quad +$

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5. For what values of x is the function $f(x) = x^4(x-2)^3$ increasing? (10)

$f'(x) = x^4\, 3(x-2)^2 + (x-2)^3 4x^3 = 0$

$x^3(x-2)^2[3x + 4(x-2)] = 0$

$x^3(x-2)^2[7x-8] = 0$

$x = 0, 2, 8/7$

$+ \quad - \quad + \quad +$

$0 \quad 8/7 \quad 2$

$\boxed{(-\infty, 0] \quad (8/7, \infty)}$

$f'(1) = + + -$

$f'(3) = + + +$

$f'(1.5) = + + +$

$f'(-1) = - + - \quad +$

$\frac{1}{2}$

6. The points x = 0 and -2 are the critical points of $f(x) = 3x^4 + 8x^3$. Test each point for local maximum, minimum, or neither. (10)

$f'(x) = 12x^3 + 24x^2$

$12x^2(x+2)$

$x = 0, x = -2$ ✓

$f''(0) = 0$? neither

$- \quad + \quad +$

$-2 \quad 0$

$f''(x) = 36x^2 + 48x$

$= 12x(3x+4)$

$f''(-2) = + \quad \cup$

rel min

f'

$\frac{1}{2}$

7. Find the derivative dy/dx when $x^3y + xy^2 + y = 4$. (10)

$x^3\frac{dy}{dx} + y3x^2 + x2y\frac{dy}{dx} + y^2 + \frac{dy}{dx} = 0$

$(x^3 + 2xy + 1)\frac{dy}{dx} = -y^2 - 3x^2y$

$\frac{dy}{dx} = \frac{-y^2 - 3x^2y}{x^3 + 2xy + 1}$

$\frac{1}{2}$

8. Find the equation of the straight line which is tangent to the curve $x^2 + y^3 = 2$ at the point (1,1). (10)

$2x + 3y^2\frac{dy}{dx} = 0$

$2 + 3\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{2}{3}$

$y - 1 = -\frac{2}{3}(x-1)$

$y = -\frac{2}{3}x + \frac{2}{3} + 1$

$\boxed{y = -\frac{2}{3}x + \frac{5}{3}}$

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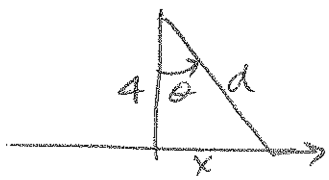
9. A radar antenna is located 4 miles from a straight shoreline. An airplane is flying along the shoreline and the radar is fixed on it. When the distance between the radar and the plane is 5 miles, the antenna is rotating at the instantaneous rate of 12 radians per hour. How fast is the the plane flying? (10)

when $d = 5$, $\frac{d\theta}{dt} = 12$ Find $\frac{dx}{dt}$



$\frac{x}{4} = \tan\theta$ $x = 4\tan\theta$ $\frac{dx}{dt} = 4\sec^2\theta \frac{d\theta}{dt}$

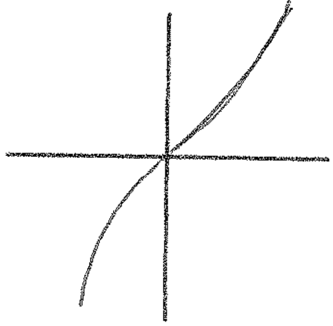
$\frac{dx}{dt} = \cancel{4} \cdot \frac{25}{\cancel{16}} \cdot \cancel{12} = 75$ mph.

$\sec\theta = \frac{5}{4}$

5 got all
3 more.
8

10. Draw the graph of a function for which $f(0) = 0$, $f'(x) > 0$ for all x, and $f''(x) > 0$ for $x > 0$ and $f''(x) < 0$ for $x < 0$. (5)

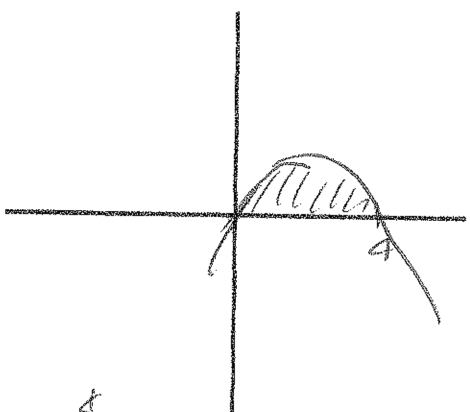
most



11. Find the center of gravity of the region bounded by the curves $y = 0$ and $y = 4x - x^2$. Sketch the region and plot the point. (15)

$\frac{1}{4}$

$x(4-x)$



$$\text{area} = \int_0^4 4x - x^2\,dx = \frac{4x^2}{2} - \frac{x^3}{3}\Big|_0^4 = 2 \cdot 16 - \frac{4^3}{3} = 32 - \frac{64}{3} = \frac{32}{3}$$

$$M_{x=0} = \int_0^4 \cancel{2\pi} x\,(4x - x^2)\,dx = 2\pi \int_0^4 4x^2 - x^3\,dx = \cancel{2\pi}\left(\frac{4x^3}{3} - \frac{x^4}{4}\right)\Big|_0^4 = \cancel{2\pi}\left(\frac{256}{3} - 64\right) = \cancel{2\pi}\left(\frac{64}{3}\right)$$

$$M_{y=0} = \int_0^4 \frac{1}{2}(4x - x^2)^2\,dx = \frac{1}{2}\int_0^4 16x^2 - 8x^3 + x^4\,dx = \frac{1}{2}\left[\frac{16x^3}{3} - \frac{8x^4}{4} + \frac{x^5}{5}\right]\Big|_0^4 = \frac{1}{2}\left[\frac{1024}{3} - 512 + \frac{1024}{5}\right]$$

$= \frac{1}{2}\left[\cancel{\frac{1024}{3}} + \cancel{1024}\right] = \cancel{\frac{1}{6}(1024)}$

$= \frac{1}{2}\left[\frac{5120}{15} - \frac{7680}{15} \ldots\right] = \cancel{\ldots} = \cancel{568}\ 17\frac{1}{15}$

$$\bar{y} = \frac{\cancel{128\pi}/3}{32/3} = \frac{\cancel{1280}\ 53/3}{32/3} = \frac{\cancel{128\pi}}{31} = 6\ \frac{53}{32}$$

$$\bar{x} = \frac{\cancel{128\pi}\ 64/3}{32/3} = 2$$

1.6

$(2, \frac{53}{32} = 1.65)$

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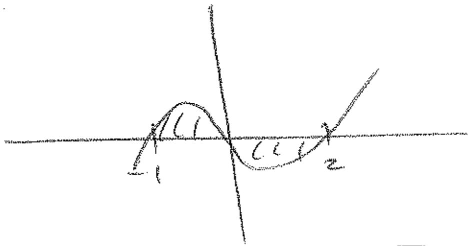
12. Find the area between the curve $y = x^3 - x^2 - 2x$ and the x-axis. (10) $\frac{4}{3}$



$x^3 - x^2 - 2x = 0$

$x(x^2 - x - 2) = 0$

$x(x-2)(x+1) = 0$

$x = 0, -1, +2$

$x=1$ $1-3=-$

$-\quad +\quad -\quad +$ (-1, 0, 2)

$\int_{-1}^{0} x^3 - x^2 - 2x\,dx$

$- \int_0^2 x^3 - x^2 - 2x\,dx$

$= \frac{x^4}{4} - \frac{x^3}{3} - \frac{2x^2}{2}\Big|_{-1}^{0} - \left(\frac{x^4}{4} - \frac{x^3}{3} - x^2\right)\Big|_0^2$

13. For the following integral: (20)

$$\int_0^2 x^3 - 2x\,dx$$

$= -\left(\frac{1}{4} + \frac{1}{3} - 1\right) - \left(\cancel{4} - \frac{8}{3} - \cancel{4}\right)$

$= -\frac{1}{4} - \frac{1}{3} + 1 + \frac{8}{3} = -\frac{1}{4} + \frac{7}{3} + 1 = \frac{-3 + 28 + 12}{12}$

$= \frac{37}{12}$

a. <u>Write out</u> (do not compute) the Riemann sum with 5 equal subintervals and the function evaluated at the right-hand endpoint. (You may do for general N.) (6)

$0\quad 1\quad 2\quad 3\quad 4\quad 5$ $\Delta x = \frac{2}{5} = .4$ $\frac{1}{2}$

$0\quad .4\quad .8\quad 1.2\quad 1.6\quad 2.0$

$((.4)^3 - 2(.4))(.4) + ((.8)^3 - 2(.8))(.4)$
$+ ((1.2)^3 - 2(1.2))(.4) + (1.6^3 - 2(1.6)).4$
$+ (2^3 - 2(2)).4$

b. Find the integral using antiderivatives.

$\int_0^2 x^3 - 2x\,dx = \frac{x^4}{4} - \frac{2x^2}{2}\Big|_0^2$

$= \frac{16}{4} - 4 = 0$

c. Find the integral as the limit of Riemann sums. $0, \cancel{\frac{4}{N}} + i(.4)$

$\sum_{i=1}^{N} \left(\left(\frac{2}{N}i\right)^3 - 2\left(\frac{2i}{N}\right)\right)\left(\frac{2}{N}\right)$

$\frac{2}{N}\left(\sum_{i=1}^{N} \frac{8i^3}{N^3} - 2\sum_{i=1}^{N}\frac{2i}{N}\right) = \frac{2}{N}\left(\frac{8}{N^3}\sum_{i=1}^{N} i^3 - \frac{4}{N}\sum_{i=1}^{N} i\right)$

$= \frac{16}{N^4}\left(\frac{N(N+1)}{2}\right)^2 - \frac{8}{N^2}\left(\frac{N(N+1)}{2}\right)$

$= 4\frac{(N+1)^2}{N^2} - 4\frac{N+1}{N} = 4\left(1 + \frac{1}{N}\right)^2 - 4\left(1 + \frac{1}{N}\right)$

$\longrightarrow 4 - 4 = 0$

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14. What is the integral for the Riemann sum: (5)

$$\sum_{j=1}^{N} (1+ c_j^2)\,\Delta x_j\ ,\ 1 = x_0 < x_1 < \cdots < x_N = 5,\ x_{j-1} \le c_j \le x_j$$

$$\int_1^5 1+x^2\,dx$$

~~[illegible]~~ many $\frac{3}{4}$

15. Find the following integrals: (30)

(6) a. $\int x^3 + \sqrt{x} + \cos x\,dx$ $\quad \frac{x^4}{4} + \frac{x^{3/2}}{3/2} + \sin x + C$

$$= \frac{x^4}{4} + \frac{2}{3}x^{3/2} + \sin x + C$$

b. $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = 2\int \sin u\,du =$

$u = \sqrt{x}$ $\quad -2\cos u + C$

$du = \frac{1}{2\sqrt{x}}\,dx$ $\quad = -2\cos\sqrt{x} + C$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

c. $\int_0^1 2x\sqrt{x+1}\,dx = \int_1^2 2(u-1)\sqrt{u}\,du = 2\int_1^2 u^{3/2} - u^{1/2}\,du$

$u = x+1$ $\quad = 2\left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right]\Big|_1^2$

$du = dx$

$x = u-1$ $\quad = 2\left[\frac{2}{5}2^{5/2} - \frac{2}{3}2^{3/2}\right] - 2\left[\frac{2}{5} - \frac{2}{3}\right]$

$x=0 \quad u=1$

$x=1 \quad u=2$

$$= 2\left[\frac{2}{5}4\sqrt{2} - \frac{2}{3}2\sqrt{2}\right] - 2\left[-\frac{4}{15}\right]$$

$$= 2\left[\frac{8}{5} - \frac{4}{3}\right]\sqrt{2} + \frac{8}{15} = \frac{2(24-40)}{15}\sqrt{2} + \frac{8}{15}$$

$$= -\frac{32}{15}\sqrt{2} + \frac{8}{15}$$

d. $\int x\sqrt{x^2 - 3}\,dx$

$u = x^2 - 3$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$ $\quad \frac{1}{2}\int \sqrt{u}\,du$

$$= \frac{1}{2}\frac{u^{3/2}}{3/2} + C = \frac{1}{3}(x^2-3)^{3/2} + C$$

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