

Name Key

1. Find  $dy/dx$  when  $x^2 + 4xy^2 - y = 3$

$$2x^2 + 4x \cdot 2y \frac{dy}{dx} + 4y^2 - \frac{dy}{dx} = 0$$

$$2x^2 + 4y^2 + \frac{dy}{dx}(8xy - 1) = 0$$

$$\frac{dy}{dx} = -\frac{2x^2 + 4y^2}{8xy - 1}$$

Ans

2. Find all critical points of the function  $f(x) = 4x^3 - 36x^2 + 7$ .

$$f'(x) = 12x^2 - 72x$$

$$= 12x(x-6)$$

$$x=0, x=6$$

abut at

3. The points  $x = 0$  and  $x = 2$  are the critical points of  $f(x) = 3x^4 - 8x^3$ .  
Test each point for local maximum, minimum or neither.

$$f'(x) = 12x^3 - 24x^2 \quad (= 12x^2(x-2))$$

$$f''(x) = 36x^2 - 48x$$

$$f''(0) = 0 ? \rightarrow f' \text{ no chg of sign} - \text{neither}$$

$$f''(2) = 36 \cdot 4 - 48 \cdot 2 = 144 - 96 > 0 \quad (\text{rel min } x=2)$$

4. For what values of  $x$  is the function  $f(x) = x^3(x+3)^3$  increasing?

$$f'(x) = x^3 \cdot 3(x+3)^2 + 3x^2(x+3)^3$$

$$= 3x^2(x+3)^2 [x + x+3]$$

$$= 3x^2(x+3)^2 (2x+3)$$

D  $x < -\frac{3}{2}$

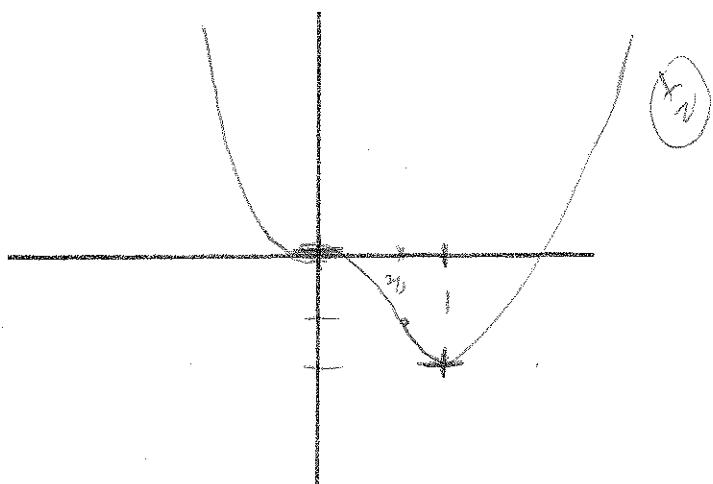
I  $x > -\frac{3}{2}$

5. For the function  $f(x) = 6x^4 - 8x^3$ , find critical points, points of inflection (if any), and sketch the graph.

$$f'(x) = 24x^3 - 24x^2$$

$$= 24x^2(x-1)$$

C.P.  $x=0, 1$



$$f''(x) = 72x^2 - 48x$$

$$= 24x(3x-2)$$

$x=0, \frac{2}{3}$

P.I.

X	Y	PI
0	0	PI
1	-2	MW
$\frac{2}{3}$	-1.18	P.I.

$$6\left(\frac{2}{3}\right)^4 - 8\left(\frac{2}{3}\right)^3$$

$$= -1.18$$

$$6\left(\frac{2}{3}\right)^4 - 8\left(\frac{2}{3}\right)^3$$

$$= -3^3$$