MATH 131 mean 18.1

Quiz ~~VII~~ 8 med 18

November 17, 1989 Name Key

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

1. Find

2 $$\sum_{i=1}^{5} (2i+1) = (3) + 5 + 7 + 9 + 11 = 35$$

2 2. Write in summation notation: 2 + 4 + 8 + 16 + 32 $\frac{1}{2}$

$$\sum_{i=1}^{5} 2^i$$

3. For $f(x) = \sqrt{x}$, write out the Riemann sum for the interval [1,4] with 3 equal subintervals in the partition. Use left endpoints. Do not add.

4 1 2 3 4 $\Delta x = 1$

$$\sqrt{1} + \sqrt{2}\cdot 1 + \sqrt{3}$$ $\frac{1}{3}$

67 4. Find the following integral by using Riemann sums: $\frac{1}{4}$

$$\int_0^3 2x^3\,dx$$

0 3/N 6/N --- 3 $\Delta x = \frac{3}{N}$

$$\sum_{i=1}^{N} 2\left(\frac{3i}{N}\right)^3 \frac{3}{N} = \frac{2(3)^4}{N^4}\sum_{i=1}^{N} i^3 = \frac{2(81)}{N^4}\left(\frac{N(N+1)}{2}\right)^2$$

$$= \frac{81}{N^2}\frac{(N+1)^2}{2} = \frac{81}{2}\left(\frac{N+1}{N}\right)^2 = \frac{81}{2}\left(1+\frac{1}{N}\right)^2$$

$$\longrightarrow \frac{81}{2} \text{ as } N \to \infty$$

15 5. Find each integral by any valid method:

a. $\int 2x^2 + \sqrt{x} - \cos x\,dx = \int 2x^2 + x^{1/2} - \cos x\,dx$

$$= \frac{2x^3}{3} + \frac{x^{3/2}}{3/2} - \sin x + C$$ $\frac{1}{2}$

$$= \frac{2}{3}x^3 + \frac{2}{3}x^{3/2} - \sin x + C$$

b. $\int_1^5 \sqrt{2x-1}\,dx = \int_1^9 \sqrt{u}\,\frac{1}{2}\,du = \frac{1}{2}\left.\frac{u^{3/2}}{3/2}\right|_1^9 = \frac{1}{3}\left(9^{3/2} - 1\right)$

$u = 2x - 1$ $x=1$ $u=+1$ $= \frac{1}{3}(27 - 1) = \frac{26}{3}$

$du = 2\,dx$ $x=5$ $u=9$ ~~$= \sqrt{3}\,4\frac{1}{3}$~~

$\frac{1}{2}du = dx$ ($\frac{1}{2}$)

c. $\int \frac{x}{(2x+1)^3}\,dx = \frac{1}{2}\int \frac{\frac{u-1}{2}}{u^3}\,du = \frac{1}{4}\int \frac{u-1}{u^3}\,du = \frac{1}{4}\int u^{-2} - u^{-3}\,du$

$u = 2x+1$ $= \frac{1}{4}\left[\frac{u^{-1}}{-1} - \frac{u^{-2}}{-2}\right] + C$ $\frac{1}{4}$

$du = 2\,dx$

$\frac{1}{2}du = dx$ $= -\frac{1}{4u} + \frac{1}{8u^2} + C$

$x = \frac{u-1}{2}$ $= -\frac{1}{4(2x+1)} + \frac{1}{8(2x+1)^2} + C$