

Name Key

(18)

1. Find the following integrals:

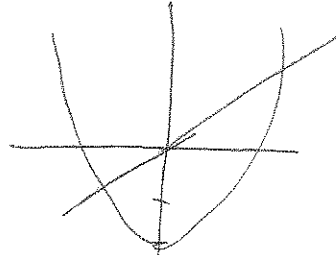
a.  $\int_1^4 \frac{x}{\sqrt{x^2+3}} dx = \frac{1}{2} \int_4^{19} u^{-1/2} du = \frac{1}{2} \frac{u^{1/2}}{1/2} \Big|_4^{19} = \sqrt{19} - \sqrt{4}$

$u = x^2 + 3$   
 $du = 2x dx$   
 $x=1 \quad u=4$   
 $x=4 \quad u=19$

b.  $\int x \sqrt{x-1} dx = \int (u+1) \sqrt{u} du = \int u^{3/2} + u^{1/2} du$

$u = x-1$   
 $x = u+1$   
 $du = dx$   
 $= \frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + C = \frac{2}{5} (x-1)^{5/2} + \frac{2}{3} (x-1)^{3/2} + C$

(6) 2. Find the area of the region bounded by the curves  $y = x^2 - 2$  and  $y = x$ .

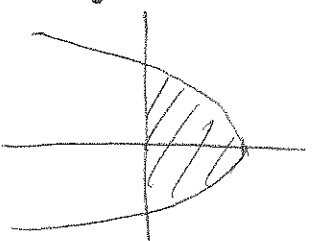


$x^2 - 2 = x$   
 $x^2 - x - 2 = 0$   
 $(x-2)(x+1) = 0$   
 $x = -1, 2$

$\int_{-1}^2 x - (x^2 - 2) dx$   
 $= \int_{-1}^2 -x^2 + x + 2 dx = -\frac{x^3}{3} + \frac{x^2}{2} + 2x \Big|_{-1}^2$   
 $= -\frac{8}{3} + \frac{4}{2} + 4 - \left(-\frac{1}{3} + \frac{1}{2} - 2\right) = -\frac{8}{3} + 6 + 2 - \frac{1}{3} + 2 = 4 - \frac{2}{3} = \frac{10}{3}$

many did volume

(6) 3. Find the area of the region bounded by the curve  $x + y^2 = 4$  and the y-axis.



$x = -y^2 + 4$   
 $\int_{-2}^2 -y^2 + 4 dx = -\frac{y^3}{3} + 4y \Big|_{-2}^2 = -\frac{8}{3} + 8 - \left(-\frac{8}{3} - 8\right) = \frac{32}{3}$

or  $2 \int_0^4 \sqrt{4-x} dx = 2 \frac{(4-x)^{3/2}}{3/2} \Big|_0^4 = \frac{32}{3}$

(6) 3. (Set up only!)  
 a. Find the volume of the solid formed by rotating about the y-axis the region bounded by  $y = \sin x$  and the x-axis for  $0 \leq x \leq \pi$ .  
 b. What is the volume if a hole of radius 1 is removed from the center?

a.  $\int_0^\pi 2\pi x \sin x dx$

b.  $\int_1^\pi 2\pi x \sin x dx$

1/3 did x-axis

32/3

of solid