$\bar{x}$ 76.3 med 78

Short. Started leaving at :36-38

many working on back page in 20 min

MATH 131
Test I
September 29, 1989 Name KEY

1. Complete the following DEFinition:
The function f is <u>continuous at the point x = a</u> if and only if (5)

must

(a) $f(a)$ is defined

(b) $\lim_{x \to a} f(x)$ exists

and (c) $\lim_{x \to a} f(x) = f(a)$

Took me 8-9 min

add a "word" problem

2. Suppose that H(x) = f(g(x)), and g(1) = 3, f(1) = 2, f(3) = 7, g(2) = -4, g(3) = 8, and f′(1) = -5, f′(2) = .5, f′(3) = 6, g′(1) = 9, g′(2) = .7, g′(3) = -10. Find: (6)

a. H(1) $= f(g(1)) = f(3) = 7$

$f'(9)$ add

many

b. H′(1). $= f'(g(1))\, g'(1) = f'(3)\, g'(1) = 6 \cdot 9 = 54$

(some $-45 = f'(1) g'(1)$)

1

3. Using only the DEFinition (any form) find the derivative of (8)

$f(x) = 3\sqrt{x} - 2$

[For possible half credit you may find f′(4) instead.] Check your answer.

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{3\sqrt{x+\Delta x} - 2 - (3\sqrt{x} - 2)}{\Delta x} = \frac{3\sqrt{x+\Delta x} - 3\sqrt{x}}{\Delta x}$$

$\frac{1}{2}$

$$= 3\,\frac{(\sqrt{x+\Delta x} - \sqrt{x})}{\Delta x}\,\frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{3}{\Delta x}\,\frac{(\cancel{x} + \Delta x - \cancel{x})}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{3}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{3}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{3}{\sqrt{x} + \sqrt{x}} = \frac{3}{2\sqrt{x}}$$

2:20

4. Find the derivative of each of the following using formulas: (35)

a. $f(x) = 2x^3 - \sqrt{x} + \frac{2}{x^4} - \sin x$ $= 2x^3 - x^{1/2} + 2x^{-4} - \sin x$

$$f'(x) = 6x^2 - \tfrac{1}{2}x^{-1/2} - 8x^{-5} - \cos x$$

b. $y = x^2 \cos x$ $y' = x^2(-\sin x) + \cos x\,(2x)$

all but 5

3

33

c. $f(x) = \dfrac{\cos x}{\sin x}$

$$f'(x) = \frac{\sin x(-\sin x) - \cos x \cos x}{\sin^2 x}$$
$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

most

d. $y = \sqrt{2x^3 - x} = (2x^3 - x)^{1/2}$

all but 2

$$y' = \tfrac{1}{2}(2x^3 - x)^{-1/2}(6x^2 - 1)$$
$$= \frac{6x^2 - 1}{2\sqrt{2x^3 - x}}$$

e. $y = \sin(x^2 - 2)$

all but 7

$$y' = \cos(x^2 - 2)(2x)$$

5. Find the following limits ($\pm\infty$ allowed), or say they do not exist. (18) 4 ← fewer pts

a. $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3} = \lim\limits_{x \to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = \lim\limits_{x \to 3}(x+3) = 6$

$\frac{0}{0}$

all but 1

b. $\lim\limits_{x \to \infty} \dfrac{1 + 2x^2}{x^2 + 3} = \lim\limits_{x \to \infty} \dfrac{1/x^2 + 2}{1 + 3/x^2} = 2$

all but 6

c. $\lim\limits_{x \to -4^+} \dfrac{x}{(x + 4)^2} = -\infty$

1/2

6. For what values is the following function not continuous? graph it (5) 4:45

1/2

$$f(x) = \begin{cases} \dfrac{1}{x}, & x < 0 \\ 3, & 0 \le x \le 1 \\ x^2 + 3, & x > 1 \end{cases}$$

$x = 0$ no limit

$x = 1$ $\lim\limits_{x \to 1} x^2 + 3 = 4$

left lim = 3

$(0, 1)$

44

5:15

7. It costs $1000 a day overhead to run a plant, and $30 direct cost to manufacture each item produced. (8)

a. Write a function for the total cost to produce x items in a day.

$f(x) = 1000 + 30x$

all part 6

b. What is the marginal total cost?

$f'(x) = 30$

8. The graphs below give z = f(u), and u=g(t). Find (and give units): (15)

a. $(f\circ g)(2)$ $= f(g(2)) = f(1.8) = 0$ lbs

b. $g'(-2)$ $(-2, -.8)$ $(0, 2.8)$ $\frac{2.8-(-.8)}{0-(-2)} = \frac{3.6}{2} = 1.8$ mph

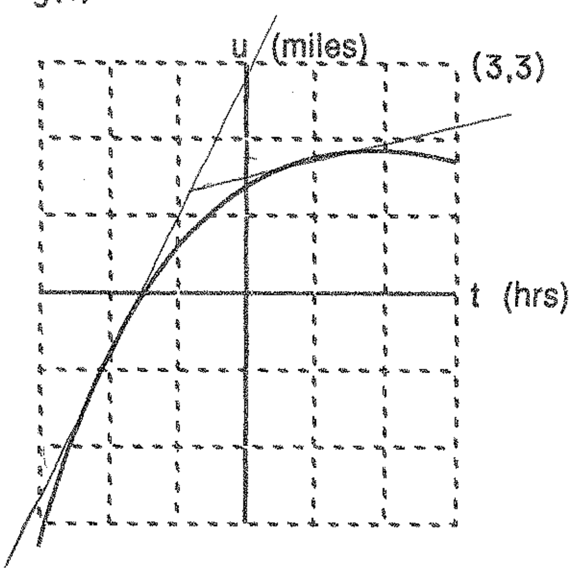
c. $\frac{dz}{dt}$, for t = 1. $\frac{dz}{du}\cdot\frac{du}{dt} = \cancel{-18}\ -2.16$ lb/hr $(.7)$ $(1, 1.8)$ $(3, 2.2)$ $\frac{2.2-(1.8)}{2} = \frac{.4}{2} = .2$

at u = 1.8 at t = 1 $(1.8, 0)$ $(0, 20)$ $\frac{20}{-1.8} = -11.1$ lbs/mi.

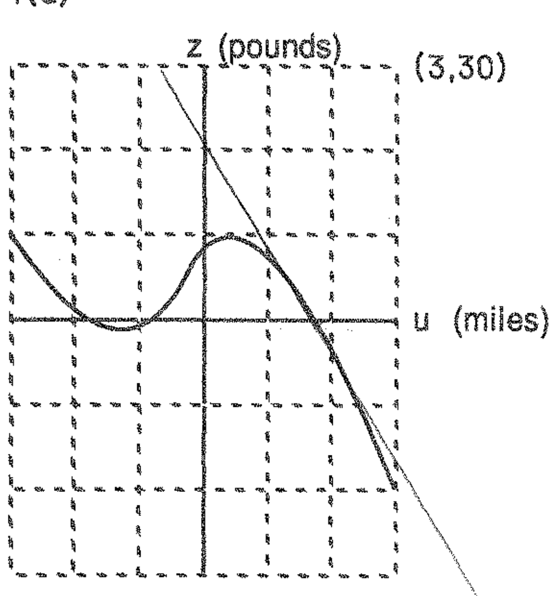
d. An object is travelling so that its position is given by g(t) at time t. What is its velocity when t = 1?

$\frac{du}{dt} = .2$ mi/hr.

g(t)



f(u)



8-7

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