Time OK

MATH 131
Test II
November 3, 1989 Name Key

1. Find all the critical points of $f(x) = x - \sqrt{x}$. $= x - x^{1/2}$ (8) fewer most

$f'(x) = 1 - \frac{1}{2}x^{-1/2} = 0$ $x = \frac{1}{4}$

$1 - \frac{1}{2\sqrt{x}} = 0$ $\boxed{\{0, \frac{1}{4}\}}$

$2\sqrt{x} = 1$ $\sqrt{x} = \frac{1}{2}$ not really (EP)

2. Find local (relative) maximum and/or minimum points (if any) for the function $f(x) = x^5 - 5x^4 + 3$. Justify. (10) $\frac{1}{2}$

$f'(x) = 5x^4 - 20x^3 = 0$ $f''(0) = 0$

$5x^3(x-4) = 0$ $f''(4) = 20(4^3) - 60(4^2)$

$x = 0, x = 4$ + − + (0, 4) $= 20(64) - 60(16)$

$= 1280 - 960 > 0$ MIN at 4

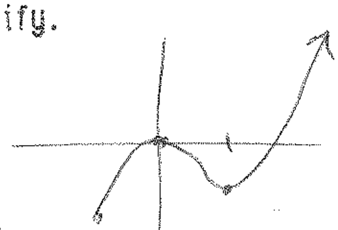
$f''(x) = 20x^3 - 60x^2$ at 0 f' ↗↘ MAX

3. Find the (absolute) maximum and minimum values (if any) of the function $f(x) = x^3 - 3x^2$ for $x \geq -2$. Justify. (10)

$f'(x) = 3x^2 - 6x = 0$

$3x(x-2)$

$x = 0, 2$



↗ ↘ ↗ (0, 2)

x	y
−2	$-8-12=-20$
0	0
2	$8-12=-4$

MAX NONE

MIN −20

4. A store knows it can sell 400 television sets in a year at a price of \$150. It also knows that for every dollar it raises the price, it will sell 2 fewer sets. What price will maximize the total revenue from the sale of these sets in a year? (10)

$R = pq$ $p = 150$ $q = 400$

$p \geq 150$ $q = 400 - 2(p-150)$

$= 400 - 2p + 300$

$= 700 - 2p$

$R = p(700-2p)$

$= 700p - 2p^2$ $0 \leq p \leq 350$

$\frac{dR}{dp} = 700 - 4p = 0$ $p = \frac{700}{4}$

$= 175$

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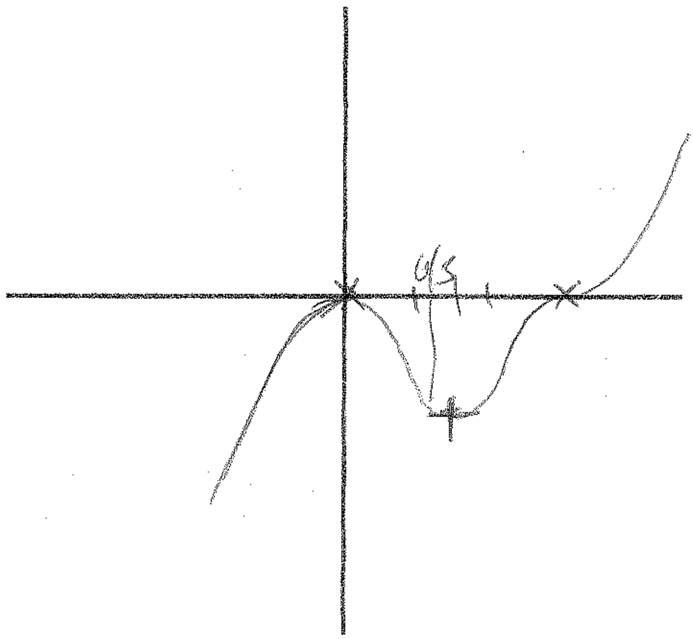
5. $f(x) = x^2(x-3)^3$ (17)

a. For what values of x is the function increasing? Decreasing?

b. Sketch the graph.



$f'(x) = x^2\,3(x-3)^2 + 2x(x-3)^3$

$= x(x-3)^2[3x + 2(x-3)]$

$= 3x(x-3)^2(5x-6)$

$x = 0,\ 6/5,\ 3$

$-\,-\qquad +\,-\qquad +\,+$

$(+)\quad 0 \quad (-) \quad 6/5 \quad (+)$

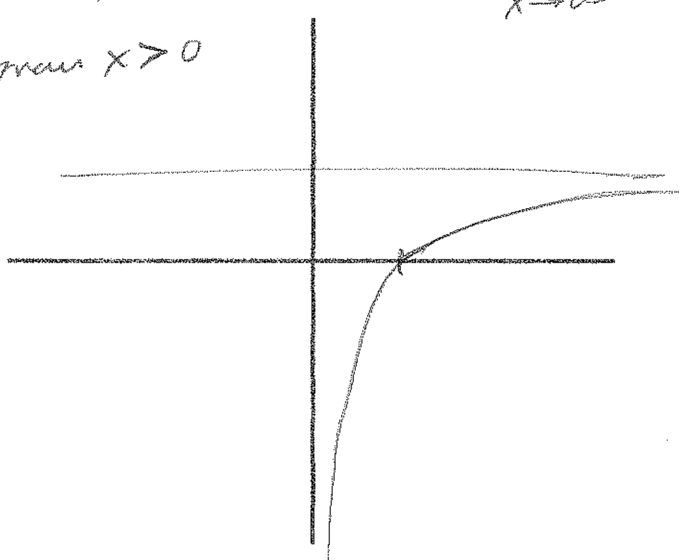
I $\;x \ge 6/5 \quad x \le 0$

D $\;0 \le x \le 6/5$

6. Sketch the graph of (10)

$y = \dfrac{\sqrt{x} - 1}{\sqrt{x}}$

Domain $x > 0$



$\lim\limits_{x\to\infty} \dfrac{1 - 1/\sqrt{x}}{1} = 1 \qquad y = 1$

$y = 1 - x^{-1/2}$

$\dfrac{dy}{dx} = -\left(-\tfrac{1}{2}\right)x^{-3/2}$

$= \dfrac{1}{2x^{3/2}}$

$\lim\limits_{x\to 0^+} \dfrac{\sqrt{x}-1}{\sqrt{x}} = -\infty$

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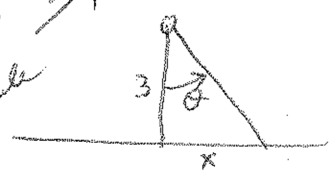
7. The light in a lighthouse is revolving at the rate of 2 revolutions per minute. The lighthouse is 3 miles from shore. At what rate is the light beam travelling along the shoreline when the light beam is perpendicular to the shore? (10)

caused trouble



$\frac{x}{3} = \tan\theta$ $\qquad \frac{d\theta}{dt} = 2(2\pi) = 4\pi$ rad/min

$\frac{1}{3}\frac{dx}{dt} = \sec^2\theta \frac{d\theta}{dt}$

$\theta = 0 \quad \sec 0 = 1$

$\frac{1}{3}\frac{dx}{dt} = 1 \cdot 4\pi$

4 pt

$\frac{dx}{dt} = 12\pi$ mi/min

8. Find the derivative dy/dx when $x^2y + xy^3 + x = 4$. (10)

$x^2\frac{dy}{dx} + 2xy + x\,3y^2\frac{dy}{dx} + y^3 + 1 = 0$

$x^2\frac{dy}{dx} + 3xy^2\frac{dy}{dx} = -1 - 2xy - y^3$

12 pt

$(x^2 + 3xy^2)\frac{dy}{dx} = \text{''}$

$\frac{dy}{dx} = \frac{-1 - 2xy - y^3}{x^2 + 3xy^2}$

9. Find the equation of the straight line which is tangent to the curve $x + \cos y = 3$ at the point (2,0). (10)

sin y

$1 \mp \sin y \frac{dy}{dx} = 0$

3 pt

$1 - \sin 0 \frac{dy}{dx} = 0$

vert $\qquad x = 2$

10. Draw the graph of a function for which $f(0) = 0$, $f'(x) < 0$ for all x, and $f''(x) > 0$ for $x > 0$ and $f''(x) < 0$ for $x < 0$. (5)

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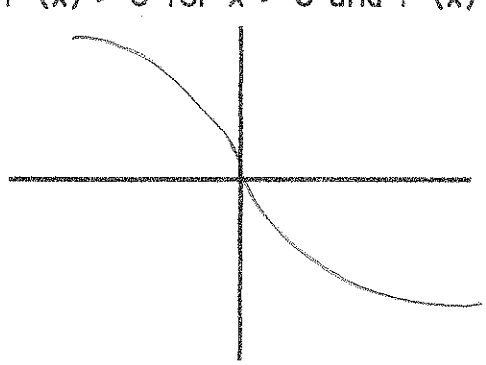

$\frac{1}{2}$

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