162 possible $\bar{x} = 111$ First at 1:15 (always left early) time OK she

med = 125 $\frac{1}{2}$ about 1:30-45

3 or 4 left at 2:10

MATH 131
Final Exam
December 16, 1992

Name Key

S-U? ___ Final Exam Double Weight? ______

1 Sheet of notes allowed

1. Find the derivative of: (20)

$\frac{16}{27}$ most close

a. $x^4 \cos x + \csc 4x$

$$x^4(-\sin x) + 4x^3\cos x - \csc 4x \cot 4x \cdot 4$$

$$-x^4\sin x + 4x^3\cos x - 4\csc 4x \cot 4x$$

b. $\dfrac{\sin x}{\sqrt{x} - x}$

20 most close

$$\frac{(\sqrt{x}-x)\cos x - \sin x\left(\frac{1}{2\sqrt{x}} - 1\right)}{(\sqrt{x}-x)^2}$$

c. $\sqrt{1 + e^{x^2-2}}$

13 many fell off (chain rule)

$$\frac{1}{2}\left(1+e^{x^2-2}\right)^{-1/2} e^{x^2-2}(2x)$$

$$\frac{xe^{x^2-2}}{\sqrt{1+e^{x^2-2}}}$$

2. The acceleration due to gravity is -32 ft/sec^2. If an object is shot upward with an initial velocity of 128 ft/sec., how high does it go? (10)

14

$a = -32$

$v = -32t + C$

$128 = C$

$v = -32t + 128$

$s = -16t^2 + 128t + C$

$0 = 0 + C$

$s = -16t^2 + 128t$

$v = 0$

$-32t + 128 = 0$

$t = 4$

$s = -16(4)^2 + 128(4)$

$= -256 + 512$

$= 256$ ft

3. Find the equation of the straight line which is tangent to the curve $x^3 + y^2 = 3$ at the point $(-1,2)$. (10)

$3x^2 + 2y\frac{dy}{dx} = 0$

$3 + 4\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{3}{4}$

$y - 2 = -\frac{3}{4}(x+1)$

$y - 2 = -\frac{3}{4}x - \frac{3}{4}$

$y = -\frac{3}{4}x + \frac{5}{4}$

TW

4. Find: (8) 5

$$\lim_{x \to -3} \frac{x^2 - 9}{x + 3} = \lim_{x \to -3} \frac{(x+3)(x-3)}{x+3} = \lim_{x \to -3} (x-3) = -3-3$$

$$= -6$$

18

(others $= \infty$)

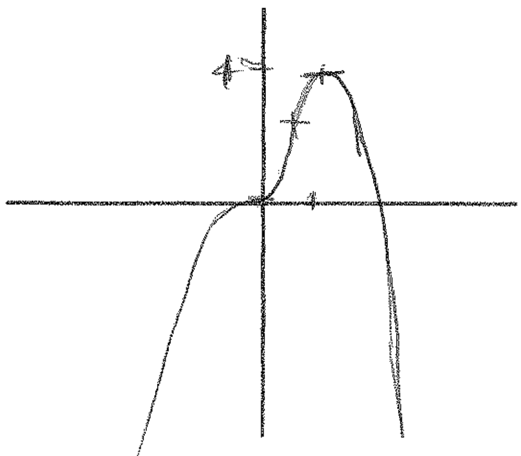
5. Sketch the graph of $f(x) = 4x^3 - 3x^4$, and find (15)

a. coordinates (if any) of critical points

b. coordinates (if any) of points of inflection

c. intervals(s) for which the graph is concavedown.

12



$f'(x) = 12x^2 - 12x^3$

$= 12x^2(1-x)$

$x=0, x=1$

ⓐ C.P. $(0,0)$ $(1,1)$

$f''(x) = 24x - 36x^2$

$= 12x(2-3x)$

$x = \frac{2}{3}$

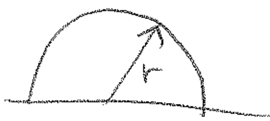
$4\left(\frac{8}{27}\right) - 3\frac{16}{81}$

ⓑ $\left(\frac{2}{3}, \frac{16}{27}\right)$ $= \frac{32-16}{27} = \frac{16}{27}$

ⓒ $(-\infty, 0)$, $\left(\frac{2}{3}, \infty\right)$

~~23~~ 20

6. A semi-circular oil spill is spreading from an oil tanker beached on the coast of Spain. When the oil slick has spread 10 miles out, it is observed to be spreading at the rate of 2 miles per day. At what rate is the area of the oil spill changing? Extra credit: At what rate is the oil leaking from the tanker? (Suppose that 100 barrels of oil will cover one square mile.) (10)



$A = \frac{1}{2}\pi r^2$

$\frac{dA}{dt} = \frac{1}{2}\pi 2r \frac{dr}{dt}$

$\frac{dr}{dt} = 2$ mi/day

$\frac{dA}{dt} = \pi(10)(2) = 20\pi$ sq mi/day

xc

$20\pi(200)$

$= 2000\pi$ b/day

all but 5 close

7. Find the area between the curves $y = 2x^4 - x^2$ and $y = x^2$. (10)

13

6 close

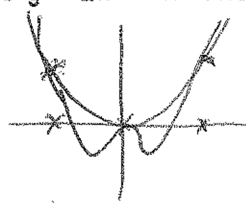
wrong space

$2x^4 - x^2 = x^2$

$2x^4 - 2x^2 = 0$

$2x^2(x^2-1) = 0$

$x = 0, \pm 1$



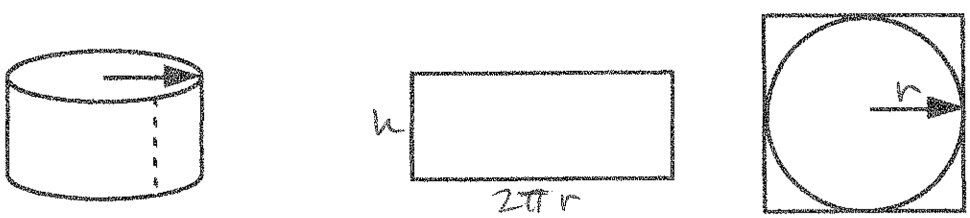
$\int_{-1}^{1} x^2 - (2x^4 - x^2)\,dx$

(some got ½ of answer)

$= \int_{-1}^{1} 2x^2 - 2x^4\,dx = \left.\frac{2x^3}{3} - \frac{2x^5}{5}\right|_{-1}^{1} = \frac{2}{3} - \frac{2}{5} - \left(-\frac{2}{3} + \frac{2}{5}\right)$

$= \frac{4}{15} + \frac{4}{15} = \boxed{\frac{8}{15}}$

8. A cylindrical box with open top is to hold 4 ft³. The circular bottom will be cut from a square piece, the rest will be waste, and the side is a rectangular piece rolled up. What dimensions will use the least amount of total material. (10)



4

2 close

$V = \pi r^2 h$

$4 = \pi r^2 h$ ———— $h = \frac{4}{\pi} r^{-2}$

$M = 2\pi r h + (2r)^2$

$= 2\pi r \frac{4}{\pi} r^{-2} + 4r^2$

$= 8r^{-1} + 4r^2$

$\frac{dM}{dr} = -8r^{-2} + 8r = 0$

$\frac{8(-1+r^3)}{r^2} = 0$

3

$\boxed{r = 1 \quad h = \frac{4}{\pi}}$

/30

9. Let the function f be given by (22)

$$f(x) = \begin{cases} -3, & x \le 0 \\ \dfrac{x-1}{x^2}, & x > 0 \end{cases}$$

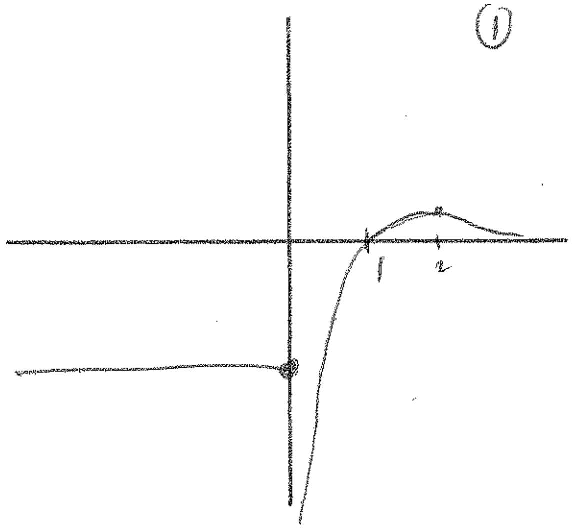
a. $\lim_{x \to 0^-} f(x) =$ -3

b. $\lim_{x \to 0^+} f(x) =$ $-\infty$

3 au c. $\lim_{x \to \infty} f(x) =$ $\lim_{x\to\infty} \frac{\frac{1}{x} - \frac{1}{x^2}}{} = 0$

$\sqrt{2}$ d. $\lim_{x \to -\infty} f(x) =$ -3

d. Sketch the graph of f. Include zeros, asymptotes, critical points.



1 $y=0$ $x=0$

$$f' = \frac{x^2(1) - (x-1)(2x)}{x^4}$$

$$= \frac{x^2 - 2x^2 + 2x}{x^4}$$

$$= \frac{-x^2+2x}{x^2} = \frac{x(-x+2)}{x^2}$$

$\cancel{x^2 = 2}$

$\cancel{x = \sqrt{2}}$

C.P $\left(\cancel{\sqrt{2}}, \frac{\sqrt{2}-1}{2}\right)$

$\left(2, \frac{1}{4}\right)$ 10 all

22

10. Find the following integrals: (20)

a. $\int \frac{4}{x} - \frac{2}{\sqrt{x}} + \sin x \, dx$ $= \int 4x^{-1} - 2x^{-1/2} + \sin x \, dx$ 11

$= +4 \ln x - 2 \frac{x^{1/2}}{1/2} - \cos x + C$

$= +4 \ln x - 4\sqrt{x} - \cos x + C$

b. $\int x \sqrt{x^2 - 3} \, dx$ $= \int \sqrt{u} \, \frac{1}{2} du = \frac{1}{2} \frac{u^{3/2}}{3/2} + C$ 17 other way off

$u = x^2 - 3$

$du = 2x \, dx$ $= \frac{1}{3}(x^2-3)^{3/2} + C$

c. $\int_0^3 2x \sqrt{x+1} \, dx$ $= \int_1^4 2(u-1)\sqrt{u} \, du = \int_1^4 2u^{3/2} - 2u^{1/2} \, du$

$u = x+1$

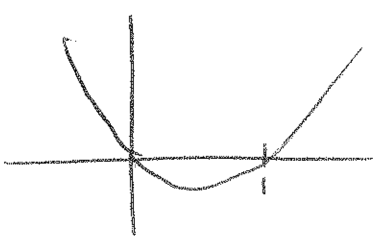
$du = dx$ $= 2\frac{u^{5/2}}{5/2} - 2\frac{u^{3/2}}{3/2} \Big|_1^4$ 9

$x = u - 1$ $= \frac{4}{5} 4^{5/2} - \frac{4}{3} 4^{3/2} - \frac{4}{5} + \frac{4}{3}$

$x = 0 \quad u = +1$

$x = 3 \quad u = 4$ $= \frac{128}{5} - \frac{32}{3} - \frac{4}{5} + \frac{4}{3} = \frac{124}{5} - \frac{28}{3}$ $= \boxed{\frac{232}{15}}$

11. Find the volume of the solid generated by revolving about the x-axis the region bounded by $y = x^2 - x$ and $y = 0$. (10)



$x^2 - x = 0$

$x(x-1) = 0$

$\int_0^1 \pi (x^2 - x)^2 \, dx$

124.3
372
−140
232

$= \int_0^1 \pi (x^4 - 2x^3 - x^2) \, dx$ 12 and 6 done

$= \int_0^1 \pi x^4 - 2\pi x^3 + \pi x^2 \, dx = \frac{\pi x^5}{5} - 2\pi \frac{x^4}{4} + \pi \frac{x^3}{3} \Big|_0^1$

$= \pi\left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3}\right) = \pi \frac{6 - 15 + 10}{30} = \frac{\pi}{30}$

30

12. The function g is defined by (20)

$$g(x) = \begin{cases} \sqrt{x}, & x \ge 4 \\ \frac{x}{2}, & x < 4 \end{cases}$$

a. What is g(4)? $\sqrt{4} = 2$

b. What is $\lim_{x \to 4^-} g(x)$? 2

c. What is $\lim_{x \to 4^+} g(x)$? 2

b. Is g continuous at 4? Justify! Yes lim $= g(4)$

(usually forget)

6 all

15 if get abs cont'd

10

c. Using only the DEFinition, find g'(4). If it doesn't exist, say so.

7 all

10 sort of

$$\lim_{h \to 0^+} \frac{g(4+h) - g(4)}{h} = \lim_{h \to 0^+} \frac{\sqrt{4+h} - 2}{h}$$

$$= \lim_{h \to 0^+} \frac{(4+h) - 4}{h(\sqrt{4+h} + 2)} = \lim_{h \to 0^+} \frac{h}{h(\sqrt{4+h} + 2)}$$

$$= \lim_{h \to 0^+} \frac{1}{\sqrt{4+h} + 2} = \frac{1}{4}$$

$$\lim_{h \to 0^-} \frac{g(4+h) - g(4)}{h} = \lim_{h \to 0^-} \frac{\frac{4+h}{2} - 2}{h} =$$

$$= \lim_{h \to 0^-} \frac{\cancel{2} + \frac{h}{2} - \cancel{2}}{h} = \lim_{h \to 0^-} \frac{1}{2} = \frac{1}{2}$$

NO.

20