MATH 131
Quiz V
October 14, 1992

Name Key

allowed 30 min
25 OK
FIRST LEFT AT 15 min
1/3 by 20
must by 25

1. Find the following derivative:

$\frac{dy}{dx}$ when $x^2 - 2x^2y^3 + 3y^2 = 3$

6

$$2x - 2\left[x^2 3y^2 \frac{dy}{dx} + y^3 2x\right] + 6y\frac{dy}{dx} = 0$$

$$(-6x^2y^2 + 6y)\frac{dy}{dx} = 4xy^3 - 2x$$

$$\frac{dy}{dx} = \frac{4xy^3 - 2x}{-6x^2y^2 + 6y} = \frac{2xy^3 - x}{-3x^2y^2 + 3y}$$

26

14 got all
3 product rule

2. Find the equation of the straight line tangent to the curve $y^3 = x^2 - 3y$ at the point (2,1).

6

$$3y^2\frac{dy}{dx} = 2x - 3\frac{dy}{dx}$$

(2,1)

$$3y' = 4 - 3y'$$

$$6y' = 4 \quad y' = 2/3$$

$$(y-1) = \frac{2}{3}(x-2) = \frac{2}{3}x - \frac{4}{3} \qquad y = \frac{2}{3}x - \frac{1}{3}$$

15 all
4 close

3. Find the linear approximation L(x) of $f(x) = 1 + 2\tan x$ at $x = 0$.

6

$$f'(x) = 2\sec^2 x$$

$$f'(0) = 2\sec^2 0 = 2$$

(0, 1) $\quad y - 1 = 2(x-0)$

$$y = 2x + 1$$

$$L(x) = 2x + 1$$

12 got all
8 close

4. A cone has base of radius 3 in. and exact height of 4 in. If the radius is known to within .1 in., what is the approximate error in volume? [Hint: $V = 1/3\ \pi r^2 \cdot 4$]

6

$$V = \frac{4}{3}\pi r^2$$

$$dV = \frac{8}{3}\pi r\, dr$$

$$dr = .1$$

$$dV = \frac{8}{3}\pi(3)(.1)$$

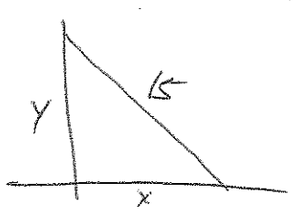
$$= 2.4\pi = 7.54 \text{ in}^3/\text{min}$$

2.51

16

5. A 15 foot ladder is leaning against a wall. The top is sliding down at the rate of 2 ft./min. when the top is 10 ft. above the ground. At that instant how fast is the bottom of the ladder moving?

6




$\frac{dy}{dt} = -2$ when $y = 10$

Find $\frac{dx}{dt}$

$$x^2 + y^2 = 15^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$x^2 = 15^2 - 10^2$$

$$= 225 - 100$$

$$= \sqrt{125} = 5\sqrt{5}$$

$$2(5\sqrt{5})\frac{dx}{dt} + 20(-2) = 0$$

$$\frac{dx}{dt} = \frac{40}{10\sqrt{5}} = \frac{4}{\sqrt{5}} \text{ ft/min}$$

1.789