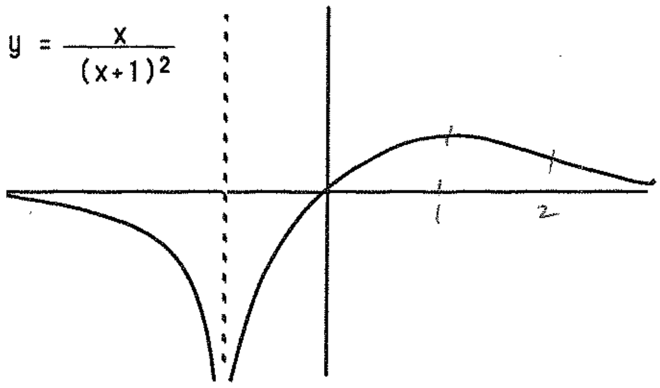
MATH 131
Quiz VI
October 22, 1992

Name *Answer*

No graphing calculators allowed!

(15) 1. This is a sketch of the graph of

$y = \dfrac{x}{(x+1)^2}$



Find (if none, say so):

a. x-y coordinates of all relative maxima and minima. *max $(1, \frac{1}{4})$ no min*

b. x-y coordinates of all points of inflection. *$(2, \frac{2}{9})$*

c. equations of all horizontal and veritical aymptotes. *$x = -1$*

d. intervals on which f is increasing. *$(-1, 1)$* *graph + part a.*

e. intervals on which f is decreasing. *$(-\infty, -1)$ $(1, \infty)$*

f. intervals on which f is concave up. *$(2, \infty)$* *graph + part b*

g. intervals on which f is concave down. *$(-\infty, -1)$ $(-1, 2)$*

$$\frac{dy}{dx} = \frac{(x+2)^2 - x\cdot 2(x+1)}{(x+1)^4} = \frac{(x+1)(x+1-2x)}{(x+1)^4} = \frac{1-x}{(x+1)^3}$$

c.p. $x = 1$ $(1, \frac{1}{4})$ rel max

no min (graph)

$$\frac{d^2y}{dx^2} = \frac{(x+1)^3(-1) - (1-x)3(x+1)^2}{(x+1)^6} = \frac{-(x+1)^2[x+1 + \overbrace{3(1-x)}^{3-3x}]}{(x+1)^6} = \frac{-(4-2x)}{(x+1)^4}$$

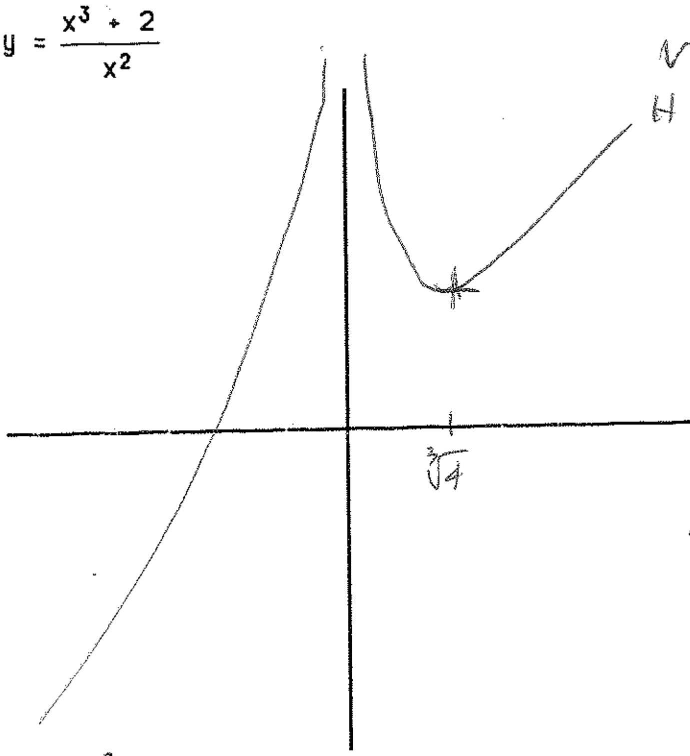
P.I. $x = 2$ $(2, \frac{2}{9})$

VA $x = -1$ $(x+1 = 0)$ HA. $y = 0$ (graph) or $\frac{x}{(x+1)^2} \approx \frac{x}{x^2} = \frac{1}{x}$

(15) 2. Carefully sketch the graph of

$y = \dfrac{x^3 + 2}{x^2}$

Name



V.A. $x = 0$

H.A. $\frac{x^3+2}{x^2} = \frac{x + \frac{2}{x^2}}{1} \xrightarrow[x\to\infty]{} \infty$

$\xrightarrow[x\to-\infty]{} -\infty$

$\lim_{x\to 0^+} \frac{x^3+2}{x^2} = +\infty$

$\lim_{x\to 0^-} \frac{x^3+2}{x^2} = +\infty$

$$\frac{dy}{dx} = \frac{x^2(3x^2) - (x^3+2)\,2x}{x^4}$$

$$= \frac{x[3x^3 - 2(x^3+2)]}{x^4}$$

$$= \frac{x^3 - 4}{x^3}$$ c.p. $x = \sqrt[3]{4}$ (only one)

also $\frac{x^3+2}{x^2} = x + \frac{2}{x^2}$ $y = x$ asymptote