allowed 27 min
First up @ 13 m
10 by 19 min
1/2 by 20
~~all but~~ 2 by 25 m

MATH 131
Quiz VIII
November 16, 1992

Name *Key*

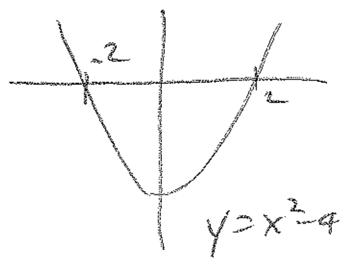
1. Express the following limit of a Riemann sum as a definite integral:

4 $\lim_{\|P\| \to \infty} \sum_{k=1}^{n} \left(c_k^2 - 3c_k\right) \Delta x_k$, where P is a partition of [1,3]. 14

$$\int_1^3 x^2 - 3x \, dx$$

2. Find the area of the region bounded by the curve $y = x^2 - 4$ and the x-axis. 13

6



$$\text{area} = -\int_{-2}^{2} x^2 - 4 \, dx$$

$$= -\left(\frac{x^3}{3} - 4x\right)\Big|_{-2}^{2} = -\left(\frac{8}{3} - 8\right) + \left(\frac{(-2)^3}{3} - (-4\cdot 2)\right)$$

$$= -\frac{8}{3} + 8 - \frac{8}{3} + 8 = 16 - \frac{16}{3}$$

$$= \frac{32}{3}$$

3. Evaluate the following intgerals:

11

5 $$\int_1^2 x^3 + \frac{1}{x^2} \, dx = \frac{x^4}{4} + \frac{x^{-1}}{-1}\Big|_1^2 = \frac{2^4}{4} - \frac{1}{2} - \left(\frac{1}{4} - 1\right)$$

$x^3 + x^{-2}$

$$= 4 - \frac{1}{2} - \frac{1}{4} + 1 = 5 - \frac{3}{4} = 4\tfrac{1}{4}$$

$$= \frac{17}{4}$$

8 $\int 3 \sec^2 x \, dx$ $\quad 3 \tan x + C$

17

5 $$\int_0^3 \frac{x}{\sqrt{x^2 + 4}} \, dx = \int_4^{13} \frac{1/2}{\sqrt{u}} \, du = \int_4^{13} \frac{1}{2} u^{-1/2} \, du = \frac{1}{2} \frac{u^{1/2}}{1/2}\Big|_4^{13}$$ 6

$u = x^2 + 4$ $\quad x = 0 \; u = 4$ $\quad = \sqrt{13} - \sqrt{4} = \boxed{\sqrt{13} - 2}$

$du = 2x \, dx$ $\quad x = 3 \; u = 13$

$\frac{1}{2} du = x \, dx$

8 $$\int x \sqrt{3x - 1} \, dx = \int x \sqrt{u} \, \tfrac{1}{3} du = \int \frac{u+1}{3} \sqrt{u} \cdot \frac{1}{3} du$$

10

$u = 3x - 1$ $$= \frac{1}{9} \int (u+1) u^{1/2} du$$

$du = 3dx$

$\frac{1}{3} du = dx$ $$= \frac{1}{9} \int u^{3/2} + u^{1/2} du$$

$3x = u + 1$

$x = \frac{u+1}{3}$ $$= \frac{1}{9}\left(\frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2}\right) + C$$

$$= \frac{1}{9}\left[\frac{2}{5}(3x-1)^{5/2} + \frac{2}{3}(3x+1)^{3/2}\right] + C$$

$$= \frac{2}{45}(3x-1)^{3/2} + \frac{2}{27}(3x+1)^{3/2} + C$$