$\bar{x}$ 78.9
m 83.5

time OK
mean time 46.1
med time 41
First left at 32
then 35
20 left bef :50

MATH 131
Test I
October 2, 1992

Name: Key

1. Find the derivatives of each of the following: (42)

a. $f(x) = 2\sqrt{x} - 3x^8 + \frac{2}{x^4} - \cos x$, $f'(x) =$

$= 2x^{1/2} - 3x^8 + 2x^{-4} - \cos x$

$f'(x) = x^{-1/2} - 24x^7 - 8x^{-5} + \sin x$

(all but 3)

b. $y = \frac{x^2 + 2}{\cot x}$

$\frac{dy}{dx} = \frac{\cot x(2x) - (x^2+2)(-\csc^2 x)}{\cot^2 x} = \frac{2x\cot x + (x^2+2)\csc^2 x}{\cot^2 x}$

11/26 got all
most got great
usually missed trig

c. $y = \sin(3x^2 - 2)$

$\frac{dy}{dx} = \cos(3x^2-1) \cdot 6x = 6x\cos(3x^2-1)$

23 got all
on face
2 made cos product

d. $y = \sec(3x^2)$

$\frac{dy}{dx} = \sec(3x^2)\tan(3x^2) \cdot 6x = 6x\sec(3x^2)\tan(3x^2)$

22 got all
now for sec tan(3x

e. $y = \sin(\tan x)$

$\frac{dy}{dx} = \cos(\tan x)\sec^2 x$

20 got all

f. $y = (5x - 3)^5$, $\frac{d^2y}{dx^2} =$

$\frac{dy}{dx} = 5(5x-3)^4 \cdot 5$
$= 25(5x-3)^4$

$\frac{d^2y}{dx^2} = 100(5x-3)^3 \cdot 5$
$= 500(5x-3)^3$

20 got all
many used prod rule on y''

g. $y = x^2 + \sqrt{x + \cos(3x - 1)}$

$= x^2 + (x + \cos(3x-1))^{1/2}$

↑ good idea

$\frac{dy}{dx} = 2x + \frac{1}{2}(x + \cos(3x-1))^{-1/2}(1 - \sin(3x-1) \cdot 3)$

9 got all
most misplaced the 3

most got close
only 3 missed eqs level

42 median 39/42

2. Find the following limits (± ∞ allowed), if they exist. (15)

a. $\lim_{x \to -\infty} \dfrac{-3x^3 - 2x + 1}{x + 4x^3}$

$\lim_{x \to -\infty} \dfrac{-3x^2 - 2 + \frac{1}{x}}{1 + 4x^2}$

$\lim_{x \to -\infty} \dfrac{-3 + \frac{2}{x^2} + \frac{1}{x^3}}{1/x^2 + 4} = -\frac{3}{4}$

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all but 3 notation problem (=)

b. $\lim_{x \to 25} \dfrac{\sqrt{x} - 5}{x^2 - 25x} \cdot \dfrac{\sqrt{x}+5}{\sqrt{x}+5}$

only 10

$\dfrac{x - 25}{x(x-25)(\sqrt{x}+5)} = \dfrac{1}{x(\sqrt{x}+5)} \to \dfrac{1}{25(5+5)} = \dfrac{1}{250}$

(those with cdc got small no, so said 0)

c. $\lim_{x \to -3^+} \dfrac{4x + 3}{x + 3}$ $= -\infty$

$x > -3$

$x + 3$ + $\quad -12+5 \cdot$ $\quad \frac{-}{+}$

14 guessed −

3. For the function (5)

$$f(x) = \begin{cases} 0 & , x \le 0 \\ -2x & , 0 < x < 2 \\ K & , x \ge 2 \end{cases}$$

a. What value of K will make the function f continuous at x = 2 (if any)? 22

$K = \lim_{x \to 2^-} -2x = -4$

b. What is the limit at x = 0 (is any)?

0

4. Using only the definition, find f'(2). If it doesn't exist, justify. (10)

$$f(x) = \begin{cases} x^2 + 2 & , x < 2 \\ 5x - 4 & , x \ge 2 \end{cases}$$

$f'(2) = \lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$ $\qquad f(2) = 5(2) - 4 = 6$

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$\lim_{h \to 0^+} \dfrac{5(2+h) - 4 - 6}{h} = \lim_{h \to 0^+} \dfrac{10 + 5h - 10}{h} = \lim_{h \to 0^+} \dfrac{5h}{h} = 5$

many close

$\lim_{h \to 0^-} \dfrac{(2+h)^2 + 2 - 6}{h} = \lim_{h \to 0^-} \dfrac{4 + 4h + h^2 - 4}{h} = \lim_{h \to 0^-} \dfrac{h(4+h)}{h}$

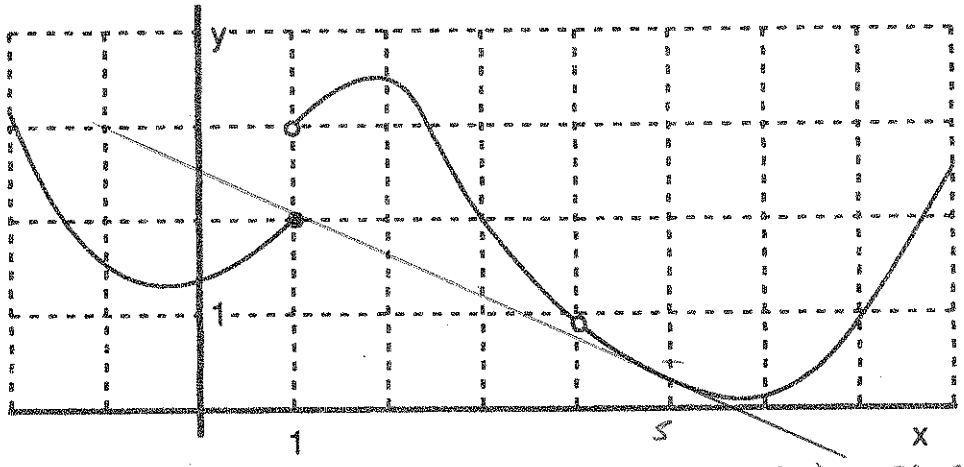
$= \lim_{h \to 0^-} 4 + h = 4$

no 2 sided limit

DNE

30

5. Suppose y = f(x) is given by the following graph. (10)



a. f(1) = 2

b. f(f(3)) = f(2) = 3.5

c. $\lim_{x \to 1^+} f(x)$ = 3

d. $\lim_{x \to 4} f(x)$ = .9

e. Estimate f'(5) (5.6, 3) (0, 2.5)

$\frac{2.5 - 3}{-5} = \frac{-.5}{5} = .1$

(5.6, 0) (0, 2.5)

$\frac{2.5}{-5.6} = -.446$

6. For what values of x is the function f not continuous? Give reasons! (8)

$$f(x) = \begin{cases} 2x, & x < -2 \\ \frac{8}{x}, & -2 \le x < 2 \\ 4, & x > 2 \end{cases}$$

x = -2 $\lim_{x \to -2^-} 2x = -4$ $\lim_{x \to -2^+} \frac{8}{x} = -4$ ✓ yes

x = 2 $\lim_{x \to 2^-} 4 =$. no f(2) = ?

x = 0 f(0) = ? not a good d

x = -2 no limit $\lim_{x \to -2^+} = \pm\infty$

7. The height (ft.) of an object above the ground is h = 200 + 32 t - 16 t^2, where t is time in seconds. (Include units with answers.) (10)

a. What is the initial velocity? up or down?

b. What is the velocity after 3 seconds? Up or down?

c. How high does the object go?

$v = 32 - 32t$

(a) 32 ft/sec up

(b) -64 ft/sec down

(c) $-32t + 32 = 0$

$t = 1$ (some unit 3)

$h = 200 + 32 - 16$

$= 216$ ft

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