

1. Find the following derivative:

$$\frac{dy}{dx} \text{ where } x^3 - xy + \cos y = 3.$$

odd May/May problem

$$3x^2 - (x \frac{dy}{dx} + y) - \sin y \frac{dy}{dx} = 0$$

$$(-x - \sin y) \frac{dy}{dx} = -3x^2 + y$$

$$\frac{dy}{dx} = \frac{-3x^2 + y}{-x - \sin y} \text{ or } \frac{3x^2 - y}{x + \sin y}$$

2. Find the linear approximation $L(x)$ of $f(x) = 1 + \sin x$ at $x = \pi$.

$$f'(x) = \cos x$$

$$y = -x + \pi + 1$$

15 all

$$f'(\pi) = -1$$

6 close

$$(\pi, 1)$$

$$L(x) = -x + \pi + 1$$

$$y - 1 = - (x - \pi)$$

$$= -x + \pi$$

3. A circle has a radius of approximately 5 in. We know the radius within $\pm .01$ in. What is the approximate error in the area?

(8)

$$A = \pi r^2$$

$$dr = .01 \quad r = 5$$

20

$$\frac{dA}{dr} = 2\pi r$$

$$dA = 2\pi(5)(.01)$$

$$= (.1)\pi = .314 \text{ in}^2$$

$$dA = 2\pi r dr$$

4. Ohm's law for electrical circuits is $V = IR$, where V is voltage (in volts), I is current (in amps.) and R is resistance (in ohms.) Suppose that the resistance is 10 ohms in our circuit (fixed). If the voltage is decreasing at the rate of 2 volts per minute, what is the rate of change of the current?

(10)

$$V = 10 I$$

13 all

$$\frac{dV}{dt} = 10 \frac{dI}{dt}$$

must use
sign on
units

$$\frac{dV}{dt} = -2$$

$$\frac{dI}{dt} = \frac{-2}{10} = -.2 \text{ amps/min}$$

all but 3
close

10 min

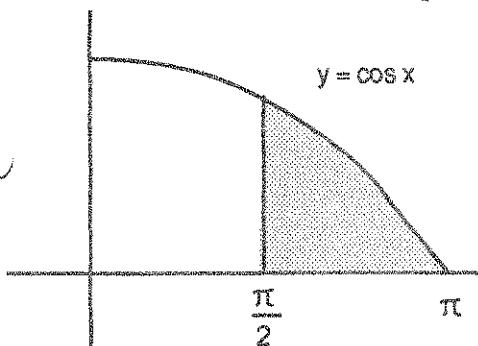
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easy phys

5. Find the area of the shaded region:

(8)

lower of two curves



$$A_{\frac{\pi}{2} \text{ to } \pi} = \sin \pi - \sin \frac{\pi}{2}$$

$$= 0 - 1 = -1$$

$$F(x) = \sin x$$

① in loops.

6. Write out the following sums:

(8)

a. $\sum_{k=1}^5 \frac{k+1}{k} = \frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5}$

$$7 \frac{17}{60}$$

all but 1

b. $\sum_{k=2}^6 (-1)^k (k^2 + 2) = (2^2 + 2) - (3^2 + 2) + (4^2 + 2) - (5^2 + 2) + (6^2 + 2)$

$$= 6 - 11 + 18 - 27 + 38 = 24$$

all but 4

7. Write in sigma notation:

(4)

$$\frac{2}{3} + \frac{2}{4} + \frac{2}{5} + \frac{2}{6} = \underbrace{\sum_{n=3}^6}_{n=3} \frac{2}{n}$$

8. Solve for y as a function of x:

(12)

a. $\frac{dy}{dx} = 4x^2 - \frac{2}{x^2} = 4x^2 - 2x^{-2}$

12 all

$$\frac{4x^3}{3} - \frac{2x^{-1}}{-1} + C$$

5 + C

b. $\frac{dy}{dx} = 2 \cos(3x)$, $y = 3$ when $x = 0$. *(start with C=0)*

16 all

$$y = \frac{2}{3} \sin 3x + C$$

$$y = \frac{2}{3} \sin 3x + 3$$

may miss off

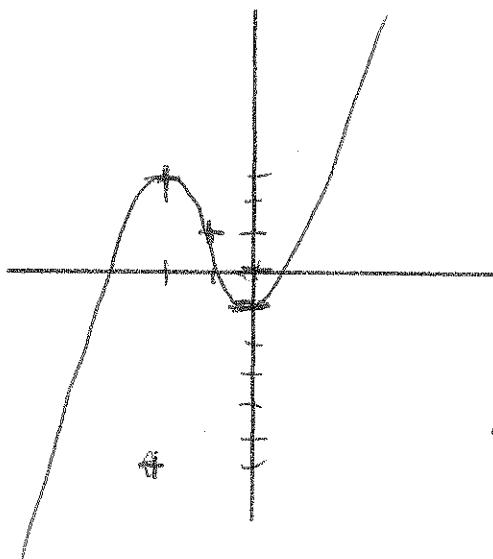
$$3 = \frac{2}{3} \sin 0 + C$$

$$C = 3$$

20 min

9. Carefully sketch the graph of $f(x) = x^3 + 3x^2 - 1$. Find x-y coordinates of critical points and points of inflection (if any.)

(15)



$$f'(x) = 3x^2 + 6x = 0$$

18 all
most close

$$3x(x+2)$$

$$x=0, -2$$

$$f''(x) = 6x + 6 = 0$$

$$x=-1$$

x	y
0	-1
-1	1
-2	3

$$\text{C.P. } (0, -1)$$

$$(-2, 3)$$

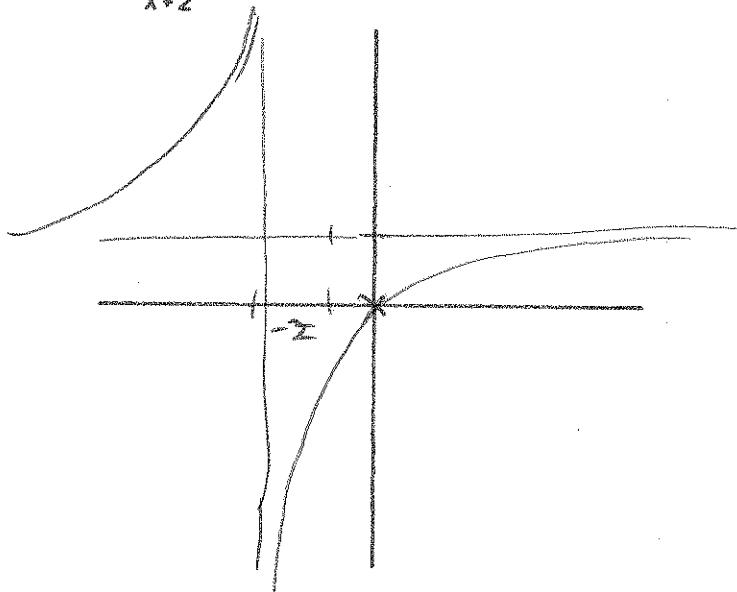
$$\text{P.I. } (-1, 1)$$

$$-8 + 12 - 1$$

10. Sketch the graph of the following. Give equations of asymptotes (if any).

(15)

$$y = \frac{x}{x+2}$$



$$\text{VA } x = -2$$

most
18 all

$$\text{HA } \frac{1}{1+\frac{2}{x}} \rightarrow 1$$

$$y = 1$$

$$\lim_{x \rightarrow -2^+} \frac{x}{x+2} = \frac{-2}{+} = -\infty$$

$$\lim_{x \rightarrow -2^-} \frac{x}{x+2} = \frac{-2}{-} = +\infty$$

$$\frac{dy}{dx} = \frac{(x+2) - x}{(x+2)^2} = \frac{2}{(x+2)^2} > 0$$

$$= 2(x+2)^{-2} \quad \text{no C.P.}$$

$$\frac{d^2y}{dx^2} = -4(x+2)^{-3} \text{ on P.I.}$$