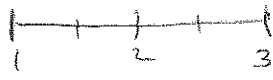


$$\sum_{k=1}^n k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

1. Find the left sum LR_4 and right sum RR_4 (with $n = 4$ subintervals) for approximating the area under the curve $f(x) = 3x^2 + 1$ for x between $x = 1$ and $x = 3$. You may use the calculator ^{most} program or do by hand.



$$LR_4 = \frac{1}{2} (3(1)^2 + 1) + \frac{1}{2} (3(\frac{3}{2})^2 + 1) + \frac{1}{2} (3(2)^2 + 1) + \frac{1}{2} (3(\frac{5}{2})^2 + 1)$$

$$= \frac{1}{2} (4) + \frac{1}{2} (\frac{27}{4} + 1) + \frac{1}{2} (13) + \frac{1}{2} (\frac{75}{4} + 1)$$

$$= \frac{1}{2} (4 + \frac{31}{4} + 13 + \frac{79}{4}) = \underline{22.25}$$

$$RR_4 = \frac{1}{2} (3(\frac{3}{2})^2 + 1) + \dots + \frac{1}{2} (3(\frac{5}{2})^2 + 1) + \frac{1}{2} (3(3)^2 + 1) = 22.25 + \frac{25}{2} = 34.25$$

2. Write out:

$$\sum_{k=2}^4 (3k + k^3) = (6 + 8) + (9 + 27) + (12 + 64) = 14 + 36 + 76 = 126$$

3. Find the antiderivative of

a. $x^3 + 7x - 1$

$$\frac{x^4}{4} + \frac{7x^2}{2} - x + C$$

b. $\sin(4x)$

$$-\frac{\cos(4x)}{4} + C$$

4. Solve for y :

$\frac{dy}{dx} = x^2 + 1, y = 3$ when $x = 1$

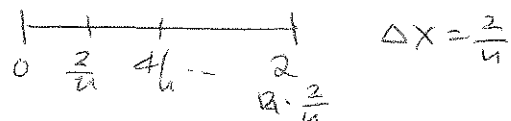
$$y = \frac{x^3}{3} + x + C$$

$$3 = \frac{1}{3} + 1 + C$$

$$C = 2 - \frac{1}{3} = \frac{5}{3}$$

5. For $f(x) = 4x^3$ for between $x = 0$ and $x = 2$:

- a. Find a formula for the right sum RR_n in terms of n .
 b. Find the limit of RR_n as $n \rightarrow \infty$.



a. $\sum_{k=1}^n \frac{2}{n} (4(\frac{2k}{n})^3)$

b. $\frac{8}{n^4} \sum_{k=1}^n (2k)^3 = \frac{64}{n^4} \sum_{k=1}^n k^3 = \frac{64}{n^4} \left[\frac{n(n+1)}{2} \right]^2$

$$= \frac{64}{n^4} \frac{(n+1)(n+1)}{4} \rightarrow 3216$$

$$4x^3 + x \Big|_1^3$$

$$27 + 1 = 28$$

$$- \frac{2}{28}$$