Little long

Bartlett @ 98 (checks) ~~Senn~~ @ 50, 51, 51, 53, 54

MATH 131
Test II
November 5, 1993

$\bar{x} = 67.8$
$m = 66.5$

Name Key

May said fair, but

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

1. Find the following integrals: (20) 5 all

a. $\int \sec^2 x =$ $\tan x + C$

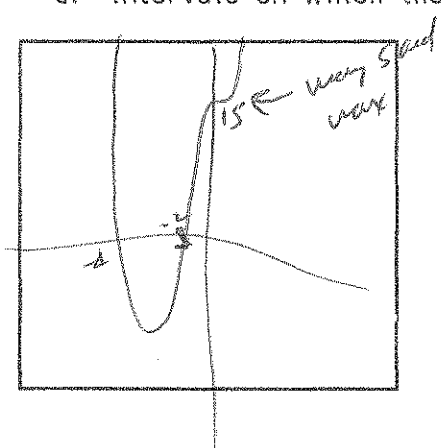
b. $\int x^3 + 3x - \sqrt{x}\ dx =$ $\frac{x^4}{4} + \frac{3x^2}{2} - \frac{x^{3/2}}{3/2} + C = \frac{x^4}{4} + \frac{3x^2}{2} - \frac{2}{3}x^{3/2} + C$

c. $\int \sin(3x)\ dx =$ $-\frac{\cos 3x}{3} + C$

d. $\int_1^2 x^2 + 3x\ dx =$ $\frac{x^3}{3} + \frac{3x^2}{2}\Big|_1^2 = \frac{8}{3} + \frac{3\cdot 4}{2} - \left(\frac{1}{3} + \frac{3}{2}\right)$

$= \frac{8}{3} + 6 - \frac{1}{3} - \frac{3}{2} = \frac{7}{3} + 6 - \frac{3}{2} = \frac{41}{6}$ $\quad \frac{14+36-9}{6}$ $\quad \frac{41}{6}$

2. Sketch the complete graph of $f(x) = x^4 + 4x^3 + 15$. Gives ranges. Find exactly (if any):
   a. x-y coordinates of relative maxima and minina. (15)
   b. x-y coordinates of points of inflection.
   c. intervals on which the function is increasing, decreasing.
   d. intervals on which the graph is concave up, concave down.



$f' = 4x^3 + 12x^2 = 4x^2(x + 3)$

$x = 0, \ -3$

(0, 15)
(-3, -12)

$f'' = 12x^2 + 24x$

$= 12x(x+2)$ $\quad x = -2$

9 must close

a MIN (-3, -12) no MAX

b PI (-2, -1) (0, 15)

c I (-3, ∞) D (-∞, -3)

d CC↑ (-∞, -2) (0, ∞)
CC↓ (-2, 0)

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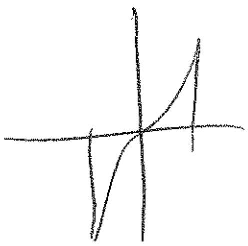
3. Find the area of the region bounded by the curve $y = x^3$, the x-axis, and $x = -2$, $x = 2$. (10)



$$\int_0^2 x^3\,dx - \int_{-2}^0 x^3\,dx$$

$$\frac{x^4}{4}\Big|_0^2 - \frac{x^4}{4}\Big|_{-2}^0$$

$$= \frac{16}{4} - 0 - 0 + \frac{16}{4} = 8$$

4 all.

Some med RPh after first ∫

4. The gravity on the moon is 1.6 m/sec/sec downwards. If an object is tossed upward with a velocity of 16 ~~m/sec.~~ (12)
   a. What is the velocity after 2 seconds?
   b. What is the maximum height attained?

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fair close

$y'' = -1.6$

$y' = -1.6t + 16 \quad t = 10$

a) $y' = 16 - 3.2 = 12.8$ m/sec

b. $y' = 0 \quad t = 10$

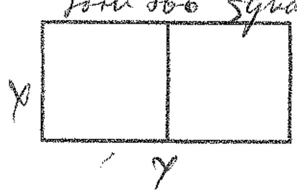
$y = -.8t^2 + 16t$

$= -.8(100) + 160 = 80$ m

bad choice, since $t = 10$ works for $y = +8t$ for

5. A corral is to be built with an inner fence as shown. The total area has to be 500 sq. ft. What dimensions will use the least amount of fence? (10)

look too square



$xy = 500 \quad y = \frac{500}{x}$

$L = 3x + 2y$

$L = 3x + \frac{1000}{x}$

$L' = 3 - 1000x^{-2} = 0$

$1000 = 3x^2$

$x^2 = \sqrt{\frac{1000}{3}}$

18.3' × 27.4'

3

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6. The edges of a cube are changing at the rate of 2 in./min. when the edges are 10 in. long. How fast is the volume changing? (10)

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

$$= 3(100) \cdot 2 = 600 \text{ in}^3/\text{min}$$

7. Find $\frac{dy}{dx}$ where $4x^2 - 3xy + y^3 - \cos y = 3$ (8)

$$8x - 3\left(x\frac{dy}{dx} + y\right) + 3y^2 \frac{dy}{dx} + \sin y \frac{dy}{dx} = 0$$

$$(-3x + 3y^2 + \sin y)\frac{dy}{dx} = -8x + 3y$$

$$\frac{dy}{dx} = \frac{-8x + 3y}{-3x + 3y^2 + \sin y}$$

8. Compute $\int_0^3 3x^2 \, dx$ (15)

not 3

a. using antiderivatives and the Fundamental Theorem (i.e. the easy way)

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$$\int_0^3 3x^2 dx = x^3 \Big|_0^3 = 27$$

b. using the <u>definition</u> (by forming RR_n and then taking the limit as $n \to \infty$).

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some close

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$[0, 3]$ $\quad \Delta x = \frac{3}{n}$

$0 \quad \frac{3}{n} \quad 2\cdot\frac{3}{n} \quad \cdots \quad n\frac{3}{n} \qquad k\frac{3}{n}$

$$\cancel{\sum_{k=1}^{n} \left(k\frac{3}{n}\right)^3 \cdot \frac{3}{n} = \frac{3^4}{n^4}\sum_{k=1}^{n} k^3 = \frac{81}{n^4}\left(\frac{n(n+1)}{2}\right)^2}$$

$$\cancel{= \frac{81}{n^2}\frac{(n+1)^2}{4} = \frac{81}{4}\frac{n+1}{n}\frac{n+1}{n} \to \frac{81}{4}}$$

$$\sum_{k=1}^{n} 3\left(k\left(\frac{3}{n}\right)\right)^2 \frac{3}{n} = \frac{81}{n^3}\sum_{k=1}^{n} k^2 = \frac{81}{n^3} \frac{n(n+1)(2n+1)}{6}$$

n^2

$$= \frac{27}{2}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) \to \frac{27}{2}\cdot 1 \cdot 2 = 27$$

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