1 hour - time OK

MATH 131
Final Exam
Part II
December 19, 1997

100

Name Key

Are you taking this class S-U? [ ]

23

1. Find the following derivatives: (13)

a. $f(x) = \cos(2x)$, find $f''(x)$.

$f'(x) = -2\sin(2x)$ 19

$f''(x) = -4\cos(2x)$

b. $y = \sec(e^x + 3)$, $\frac{dy}{dx} =$ $\sec(e^x+3)\tan(e^x+3)\,(e^x)$ 16

2. Find the derivative of the function (15)

$f(x) = \frac{x}{x+1}$

23 (2 only)

a. using formulas:

$$f'(x) = \frac{(x+1) - x}{(x+1)^2} = \frac{1}{(x+1)^2}$$

b. using only the definition:

$$\frac{f(x+h) - f(x)}{h} = \frac{1}{h}\left[\frac{x+h}{x+h+1} - \frac{x}{x+1}\right]$$

$$= \frac{1}{h}\,\frac{(x+h)(x+1) - x(x+h+1)}{(x+h+1)(x+1)}$$

$$= \frac{1}{h}\,\frac{\cancel{x^2} + \cancel{xh} + \cancel{x} + h - \cancel{x^2} - \cancel{xh} - \cancel{x}}{(x+h+1)(x+1)}$$

$$= \frac{1}{(x+h+1)(x+1)}$$

almost all of next, only 1 more

$$\lim_{h\to 0} \frac{1}{(x+h+1)(x+1)} = \frac{1}{(x+1)(x+1)} = \frac{1}{(x+1)^2}$$

28

3. Find the following limit (7)

$$\lim_{x\to 3} \frac{x-3}{x^4-9x^2}$$

$$\lim_{x\to 3} \frac{x-3}{x^2(x^2-9)} = \lim_{x\to 3} \frac{\cancel{x-3}}{x^2(\cancel{x-3})(x+3)}$$

$$= \lim_{x\to 3} \frac{1}{x^2(x+3)} = \frac{1}{9\cdot 6} = \frac{1}{54}$$

all but 4

4. Find the following integrals: (30)

a. $\int_1^4 x^2 + 2\sqrt{x}\,dx = \int_1^4 x^2 + 2x^{1/2}\,dx$

chart ok

$$= \frac{x^3}{3} + \frac{2x^{3/2}}{3/2}\Big|_1^4 = \frac{4^3}{3} + \frac{4}{3}4^{3/2} - \left[\frac{1}{3} + \frac{4}{3}\right]$$

$$= \frac{64}{3} + \frac{\cancel{128}\ 32}{3} - \frac{1}{3} - \frac{4}{3} = \cancel{\frac{61}{3}}\ \cancel{\frac{190}{3}}$$

$$\frac{91}{3} = \cancel{\ }$$

b. $\int \frac{\cos x}{\sqrt{\sin x}}\,dx = \int \frac{1}{\sqrt{u}}\,du$

$u = \sin x$

$du = \cos x\,dx$

$$= \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + C$$

$$= 2\sqrt{u} + C = 2\sqrt{\sin x} + C$$

12 all

4 ln u

c. $\int \frac{1}{3x+2}\,dx = \int \frac{1}{u}\,\frac{1}{3}\,du$

$u = 3x + 2$

$du = 3\,dx$

$$= \frac{1}{3}\ln u + C$$

$$= \frac{1}{3}\ln(3x+2) + C$$

13

d. $\int_1^4 \frac{x}{2x+1}\,dx = \int_3^9 \frac{\frac{u+1}{2}}{u}\,\frac{1}{2}\,du$

$u = 2x+1$

$du = 2\,dx$

$x = \frac{u+1}{2}$

$2\,du = dx$

$x=1 \quad u=3$

$x=4 \quad u=9$

$$= \frac{1}{4}\int_3^9 1 + \frac{1}{u}\,du = u + \ln u\Big|_3^9$$

few

$$= 9 + \ln 9 - 3 + \ln 3$$

$$= 6 + \ln 9 - \ln 3 = 6 + \ln 3$$

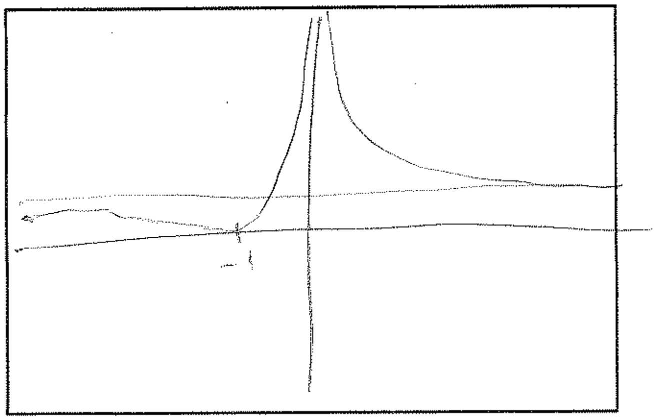
~~4.09~~ $\frac{4.801}{4}$ 1.22

37

5. For the function (15)

$$f(x) = \frac{(x+1)^2}{x^2}$$

a. Find the equations of asymptotes, if any. ← DRAM
b. Find any points of discontinuity, if any.
c. Find coordinates of all maxima, minima, and points of inflection.
d. <u>Carefully</u> sketch the graph below. Use a straight edge! Choose a proper window.
e. For what intervals is the function increasing? decreasing?
f. For what intervals is the graph concave up? down?



(a) $x = 0$

$$\lim_{x\to\infty} \frac{x^2+2x+1}{x^2} = \lim_{x\to\infty} \frac{1+2/x+1/x^2}{1} = 1$$

$y = 1$

(b) $x = 0$

(c) $$f'(x) = \frac{x^2(2(x+1)) - (x+1)^2 2x}{x^4}$$

$$= \frac{2x(x+1)[x-(x+1)]}{x^4}$$

$$= \frac{2(x+1)(-1)}{x^3}$$

$$= \frac{-2(x+1)}{x^3}$$ CP $x=-1$

$$f''(x) = \frac{x^3(-2) + 2(x+1)(3x^2)}{x^6}$$

$$= \frac{2x^2[-x+3(x+1)]}{x^6}$$

$$= \frac{2(2x+3)}{x^4}$$

C.P. $x = -\frac{3}{2}$

$(-1.5, .\overline{11})$

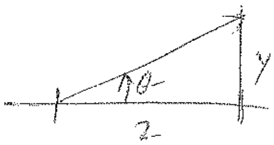
Min at $(-1, 0)$
global no max
PI $(-\frac{3}{2}, \frac{1}{9})$

e ↑ $(-1, 0)$ ↓ $(-\infty, -1)$ $(0, \infty)$

f cc ↑ $(-\frac{3}{2}, 0)$ $(0, \infty)$ $(-1, 0)$
cc ↓ $(-\infty, -\frac{3}{2})$

15

6. A rocket is launched vertically upward from a point 2 miles west of an observer. What is the speed of the rocket when the angle of elevation (from the horizontal) of the observers line of sight to the rocket is 45 degrees, and is increasing at 5 degrees per second? (10)



$$\frac{y}{2} = \tan\theta$$

$$\frac{1}{2}\frac{dy}{dt} = \sec^2\theta \, \frac{d\theta}{dt}$$

12

$$\sec^2(45°) = \left(\frac{1}{1/\sqrt{2}}\right)^2 = 2$$

$$\frac{d\theta}{dt} = 5 \cdot \frac{\pi}{180} = \frac{\pi}{36}$$

$$\frac{dy}{dt} = 2 \cdot 2 \cdot \frac{\pi}{36} = \frac{\pi}{9} \text{ mi/sec}$$

.349 mi/sec

7. To show what you know, make up a good problem that you are confident about--and was not covered in other problems--and solve it. (10)

30