

1. In the following integrals, you may use substitution.

a. $\int_1^4 x^3 + \sqrt{x} \, dx = \int_1^4 x^3 + x^{1/2} \, dx = \frac{x^4}{4} + \frac{x^{3/2}}{3/2} \Big|_1^4$ *all best 1*
 $= \frac{4^4}{4} + \frac{2 \cdot 4^{3/2}}{3} - \left(\frac{1}{4} + \frac{2}{3} \right) = 64 + \frac{16}{3} - \frac{1}{4} - \frac{2}{3}$ *8^2 / 12*

b. $\int_0^2 (2x - 3)^3 \, dx = \int_{x=0}^{x=2} u^{3/2} \frac{1}{2} \, du$ *all best 2*
 $u = 2x - 3$
 $du = 2 \, dx$
 $= \frac{u^4}{8} \Big|_{x=0}^{x=2} = \frac{(2x-3)^4}{8} \Big|_0^2 = \frac{1}{8} - \frac{81}{8} = -10$ *64 - 3 - 1 = 60; 64 - 3 - 8 = 53*

c. $\int \cos(3x) \, dx = \int \cos u \cdot \frac{1}{3} \, du = \frac{\sin u}{3} + C$ *all best 4*
 $u = 3x$
 $du = 3 \, dx$
 $\frac{1}{3} \, du = dx$
 $= \frac{\sin 3x}{3} + C$ *(all done)*

d. $\int \sin^2 x \cos x \, dx = \int u^2 \, du = \frac{u^3}{3} + C$ *all best 7*
 $u = \sin x$
 $du = \cos x \, dx$
 $= \frac{(\sin x)^3}{3} + C$

e. $\int \frac{x}{\sqrt{x+1}} \, dx$ [Hint: try $u = x+1$.] *all best 9*
 $u = x+1$
 $du = dx$
 $\int \frac{u-1}{\sqrt{u}} \, du = \int u^{1/2} - u^{-1/2} \, du = \frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} + C$ *all done*
 $= \frac{2}{3} (x+1)^{3/2} - 2(x+1)^{1/2} + C$

2. True or false: (Give reasons.)

$\int \ln x \, dx = x \ln x - x + C$
 $D(x \ln x - x) = x \cdot \frac{1}{x} + \ln x - 1 = 1 + \ln x - 1 = \ln x$
 True.

3. A force of 10 pounds is required to hold a spring 1 foot beyond its natural length. How much work is done in pulling the spring from rest to 5 feet beyond its natural length?

$F(x) = kx$ (Hooke's law)
 $F(1) = 10$
 $10 = k \cdot 1$
 $k = 10$
 $F(x) = 10x$
 $\int_0^5 10x \, dx = \frac{10x^2}{2} \Big|_0^5$
 $= 5(25) - 0$
 $= 125 \text{ ft-lbs}$

all best 9
all done
all best 10
(most done)