First @ 41, 57,

Some checking some going fast at end.

MATH 131
Test I
October 13, 1997

Name Key

1. Find the derivatives of each: (20)
   a. $f(x) = x^{20} + \sin x$

      $20x^{19} + \cos x$

      all but 1

   b. $y = (\sin x)(\cos x)$

      $\frac{dy}{dx} = \sin x(-\sin x) + \cos x(\cos x)$

      all but 3

   c. $y = (\tan x)^3$

      $\frac{dy}{dx} = 3(\tan x)^2 \sec^2 x$

      all but 7
      3 forgot chain rule

   d. $f(x) = \sec(x^2)$

      $f'(x) = \sec(x^2)\tan(x^2)\,2x$

      1 & all

2. Derive ONE of the following (as instructed in class): (10)
   a. The power formula for positive integer powers. ~~||||~~ | $\frac{1}{2}\frac{1}{2}$
   b. The product rule. $||\frac{1}{2}\frac{1}{4}$
   c. The derivative of sin x. |||

a ~~||||~~ ||| (3 for 2 or 3)
b ||
c ||||

3. There are two graphs of functions on the last page. $z = g(u)$ and $u = f(t)$. Show all work there! Assume each block is one unit. Give units of answers. (15)

a. Estimate the rate of change of u with respect to t at t = -2 hrs.

b. For approximately what value(s) of t is $f'(t) = 0$.

c. Estimate $g(f(0))$.

d. Estimate $\frac{dz}{dt}$ at t = 0.

4. For the function

$$f(x) = \frac{x-4}{x(x^2-16)}$$ (10)

a. Find $\lim_{x\to 4} f(x)$ (analytically for full credit)

5 all

6

$$\lim_{x\to 4} \frac{\cancel{x-4}}{x(x-4)(x+4)} = \lim_{x\to 4} \frac{1}{x(x+4)} = \frac{1}{4\cdot 8} = \frac{1}{32}$$

almost all

b. Is f continuous at x = 4? Explain.

4 no f(4) DNE.

most not

5. a. Using only the definition show the derivative of the function (15)

$f(x) = \sqrt{x}$ is $f'(x) = \frac{1}{2\sqrt{x}}$

b. By rewriting with exponents, verify that this is just the power formula.

c. Use part a, and the chain rule, to show that the power formula works for $x^{3/2}$.

$= (\sqrt{x})^3$ on board.

a. $$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

all last 8

9 all

9 most got a

1 b. $$D x^{1/2} = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$$

2 c. $$y = x^{3/2} = (\sqrt{x})^3$$

$$\frac{dy}{dx} = 3(\sqrt{x})^2 \frac{1}{2\sqrt{x}} = \frac{3x}{2\sqrt{x}} = \frac{3}{2}x^{-1/2}$$

2

40

6. What are the absolute (or global) maximum and minimum values for $f(x) = x^3 + x^2$ on the closed interval [-2,0]. What is it about f that guarantees these both exist? (10)

$f'(x) = 3x^2 + 2x = 0$

$x(3x+2) = 0$

$x = 0, -\frac{2}{3}$

x	y
0	0
-2/3	.148
-2	-4

max .148

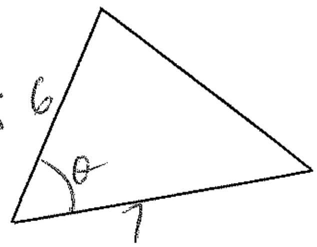
MIN -4

9 cont

13 all

7. A triangle has two sides that are 6 and 7 cm. What angle at this vertex would give the maximum area? What values of θ makes sense? [The formula for the area is (1/2) a b sin θ, where θ is the angle between the sides with lengths a and b.] (10)

Not calculus



easy to get around

$A = \frac{1}{2} \cdot 6 \cdot 7 \sin\theta \qquad 0 \le \theta \le \pi$

$= 21 \sin\theta$

$\frac{dA}{d\theta} = 21\cos\theta = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

θ	A
0	0
π/2	+
π	0

11 all (some avoided calculus)

4 close

8. Suppose we know from market research that we can sell 5000 tickets to a concert at a price of $20, for a total revenue of $100,000. We also know that for every $1 we raise the price, 50 fewer people will buy. (So if we raise the price $100, nobody will come, and we get $0.) How much can we raise the price to maximize our total revenue? (10)

x = price rise

$x = 0 \quad R = 5000(20)$

$R = (20+x)(5000-50x) \qquad 0 \le x \le 100$

$R' = (20+x)(-50) + (5000-50x)$

$= -1000 - 50x + 5000 - 50x$

$= 4000 - 100x = 0$

$x = \$40$

6 all

3 avoided calculus

/30

4 basically blank

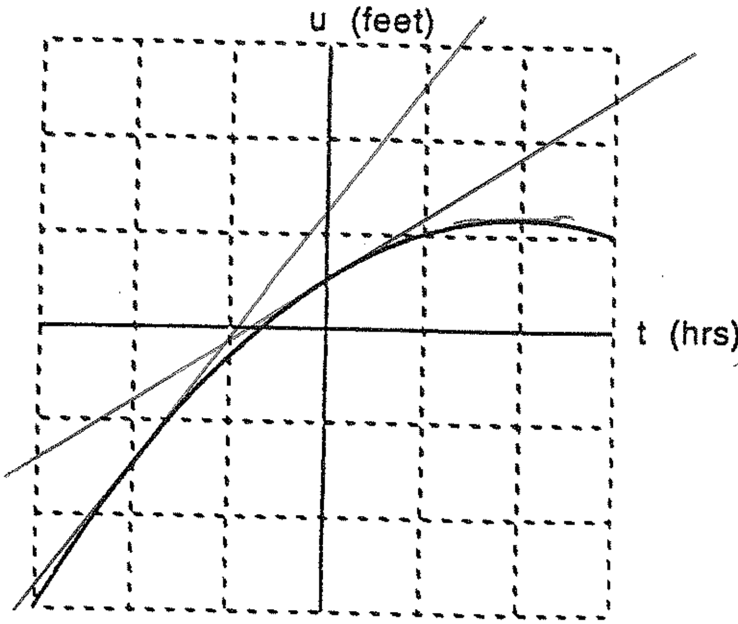
4 got all

4 a $\frac{du}{dt}$ at $t=-2$

$(-2,-1.5)$
$(1,\ 2.6)$

$\frac{2.6-(-1.5)}{1-(-2)} = \frac{4.1}{3}$

$= 1.36$ ft/hr



3

b. 2

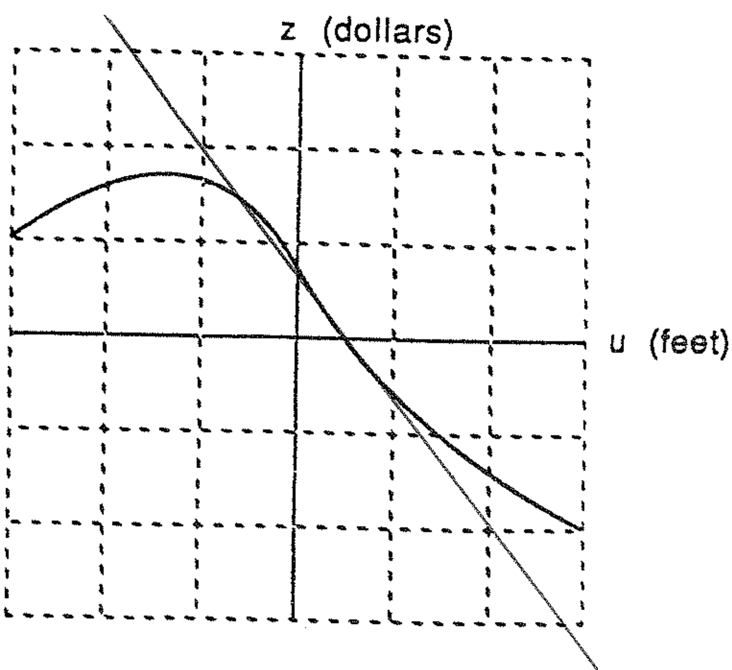
4 c. $g(f(0))$

$g(0.5)=0$

4 d. $\frac{dz}{dt}$ at $t=0$

$\frac{du}{dt}$ $(0,.5)\ (3,2.4)$

$\frac{2.4-.5}{3-0} = \frac{1.9}{3}$



$\frac{dz}{du}$ at $u=.5$

$(.5,0)\ (2.8,-3)$

$\frac{0-(-3)}{.5-2.8} = \frac{3}{-2.3}$

too close to line at 0

$\frac{dz}{dt} = \frac{dz}{du}\cdot\frac{du}{dt} = \frac{3}{-2.3}\cdot\frac{1.9}{3}$

$= \frac{1.9}{-2.3} = -.826$ \$/hr

$.63(-1.3)$