24 Long Some [illegible] on [illegible] [illegible] at 45

MATH 131H
Test II
November 17, 1997 Name ______________________

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Use calculus methods for full credit.

1. Find the indicated derivatives in each: (15) Ball

a. $f(x) = e^{3x} + (\ln x)^2$, find $f'(x)$

$$f'(x) = e^{3x} \cdot 3 + 2(\ln x)\frac{1}{x}$$

[illegible]

19

b. $f(x) = x^3 - 7\sin x$, find $f''(x)$

23

$$f'(x) = 3x^2 - 7\cos x$$
$$f''(x) = 6x + 7\sin x$$

c. $x^3 + xy^2 + 3y^3 - x = 10$, find dy/dx

18

$$3x^2 + x\,2y\frac{dy}{dx} + y^2 + 9y^2\frac{dy}{dx} - 1 = 0$$
$$(2xy + 9y^2)\frac{dy}{dx} = 1 - 3x^2 - y^2$$
$$\frac{dy}{dx} = \frac{1 - 3x^2 - y^2}{2xy + 9y^2}$$

false out [illegible]

2. Find the equation of the line tangent to the curve $4x^3 + xy + y^2 = 4$ at $(1, -1)$. (10)

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$$12x^2 + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$
$$(x + 2y)\frac{dy}{dx} = -12x^2 - y$$
$$(1 - 2)\frac{dy}{dx} = -12 + 1$$
$$\frac{dy}{dx} = 11$$

$$y + 1 = 11(x - 1)$$
$$y + 1 = 11x - 11$$
$$y = 11x - 12$$

95

3. Find the following limits: (10)

a. $\lim_{x \to \infty} \frac{3x^3 - x}{1 - x^3}$ $\lim_{x\to\infty} \frac{3 - 1/x^2}{1/x^3 - 1} = \frac{3-0}{0-1} = -3$ 22

b. $\lim_{x \to \infty} \frac{3x^3}{1 - x^4}$ $= \lim_{x\to\infty} \frac{3/x}{\frac{1}{x^4} - 1} = \frac{0}{-1} = 0$ 21

one factored down

4. Find the following indefinite integrals (antiderivatives): (15)

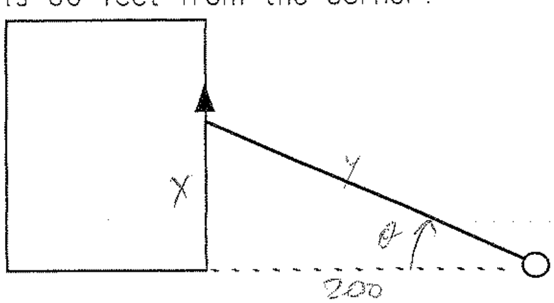
a. $\int 3x^4 - \sec^2 x \, dx$ $= \frac{3x^5}{5} - \tan x + C$ 20

↑ 5 missed this

b. $\int \sin(3x + 2) \, dx$ $= -\frac{\cos(3x+2)}{3} + C$ 13

c. $\int \frac{1}{x+1} \, dx$ $= \ln(x+1) + C$ 4

5. A search light is 200 feet from the corner of a building as shown. It is turning at one revolution every 4 minutes. How fast is the light on the wall moving when it is 50 feet from the corner? (10)



$\tan\theta = \frac{x}{200}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{200} \frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{1}{4} \cdot 2\pi = \frac{\pi}{2}$ rad/min

$x = 50$

$y = \sqrt{50^2 + 200^2}$

$= \sqrt{42500}$

$= 206.16$

$* \quad 1.0625 \cdot \frac{\pi}{2} = \frac{1}{200} \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{200\pi \; 1.0625}{\cancel{1.0625} \cdot 2} = \cancel{295.679}$ 333.79 ft/min

$\sec^2\theta = \frac{42500}{200^2} = 1.06252$

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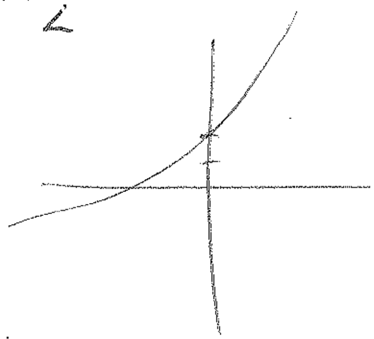
6. Draw a graph of a function f for which $f(0) = 2$, $f'(x) > 0$ all x, and $f''(x) > 0$ for $x < 0$, and $f''(x) > 0$ for $x > 0$. (5)



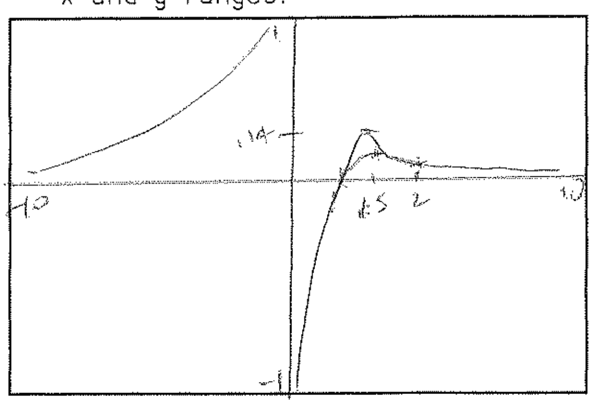
7. For the function (15)

$$f(x) = \frac{x-1}{x^3}$$

a. Find equations or coordinates (if none, say so) of local or global maxima and minima, points of inflection, and asymptotes. Plot these on the graph.
b. On what intervals is the function increasing? decreasing? concave up? concave down?
c. Carefully draw the graph in a window which shows appropriate detail. Give the x and y ranges.

6 all

c



$$f'(x) = \frac{x^3 - (x-1)3x^2}{x^6}$$

$$= \frac{x^3 - 3x^3 + 3x^2}{x^6}$$

$$= \frac{x^2[-2x+3]}{x^6}$$

$$= \frac{-2x+3}{x^4}$$

C.P.
$x = \frac{3}{2}$
$y = .148$

PI $x = 2$
$y = .125$

$$f''(x) = \frac{x^4(-2) - (-2x+3)4x^3}{x^8}$$

$$= \frac{-2x^4 + 8x^4 - 12x^3}{x^8}$$

$$= \frac{x^3(6x-12)}{x^8}$$

$$= \frac{6x-12}{x^5} \quad x = 2$$

$\lim_{x \to \infty} \frac{x-1}{x^3} = 0 \quad y = 0$

$\lim x = 0$

(a) Local Max $(\frac{3}{2}, .148)$
HA. $y = 0$
VA $x = 0$
PI $(2, .125)$

(b) I $(-\infty, 0), (0, \frac{3}{2}]$
D $[3/2, \infty)$
CC ↑ $(-\infty, 0), (2, \infty)$

(c) CC ↓ $(0, 2)$

20

8. For the function $f(x) = 3x^2$ on the interval [0,4]: (10)

a. Write the upper sum $\overline{A}_n$ (overestimate) for the area under the curve using a partition of [0,4] into n equal subintervals.

b. Find the area under the curve by finding $\lim_{n\to\infty} \overline{A}_n$.

$\Delta x = \frac{4}{n}$ $0, \frac{4}{n}, 2\frac{4}{n}, 3\frac{4}{n} \ldots n\frac{4}{n} = 4$

$A_n = 3\left(\frac{4}{n}\right)^2\frac{4}{n} + 3\left(2\cdot\frac{4}{n}\right)^2\frac{4}{n} + \cdots + 3\left(n\cdot\frac{4}{n}\right)^2\cdot\frac{4}{n}$

$= \frac{3\cdot 4^3}{n^3}\left[1 + 2^2 + 3^2 + \cdots + n^2\right] = \frac{3\cdot 4^3}{n^3}\,\frac{n(n+1)(2n+1)}{6}$

$= 32 \cdot \frac{n}{n}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)$ 5 all

b $\lim_{n\to\infty} \overline{A}_n = \cancel{3}\lim_{n\to\infty} 32\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) = 64$

about $\frac{1}{2}$ class

9. A projectile is shot straight up from the surface of the moon with a velocity of 100 m/sec. The deceleration due to gravity is about a constant 1.6 m/sec/sec. Derive an equation for the height after t sec. [Extra credit: What is the maximum height?] (10)

2

6 all

4 class

$a = \cancel{-100} = -1.6$

$v = -1.6t + C$

$100 = 0 + C$

$v = -1.6t + 100$

$h = -.8t^2 + 100t + C$

$\boxed{h = -.8t^2 + 100t}$

$v = 0$

$-1.6t + 100 = 0$

$t = \frac{100}{1.6}$

$h = .8\left(\frac{100}{1.6}\right)^2 + 100\left(\frac{100}{1.6}\right)$

3125 km.

2a