Time OK

Math 131
Test II
November 12, 1971

NAME Key

Show work for full credit!

(10) I. Using only the definition, find the derivative of $f(x) = 2x^2 + 3x - 5$.

$$\frac{f(y)-f(x)}{y-x} = \frac{2y^2+3y-5-(2x^2+3x-5)}{y-x} = \frac{2(y^2-x^2)+3(y-x)}{y-x}$$

$$= 2(y+x)+3$$

$$f'(x) = \lim_{y\to x} \frac{f(y)-f(x)}{y-x} = \lim_{y\to x} 2(y+x)+3 = 4x+3$$

(30) II. Find the derivative of each of the following functions. Show work.

1. $f(x) = 10x^3 - 3x + 2$

$$f'(x) = 30x^2 - 3$$

2. $f(x) = (x^2+3x-2)(x^4+3x^2+1)$

$$f'(x) = (x^2+3x-2)(4x^3+6x) + (2x+3)(x^4+3x^2+1)$$

3. $f(x) = \frac{x^2+x}{2x-1}$

$$f'(x) = \frac{(2x-1)(2x+1)-(x^2+x)(2)}{(2x-1)^2}$$

$$= \frac{4x^2-1-2x^2-2x}{(2x-1)^2}$$

$$= \frac{2x^2-2x-1}{(2x-1)^2}$$

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4. $f(x) = \sqrt{6x^2-1} = (6x^2-1)^{1/2}$

$f'(x) = \frac{1}{2}(6x^2-1)^{-1/2} \, 12x$

$= 6x(6x^2-1)^{-1/2}$ or $\frac{6x}{\sqrt{6x^2-1}}$

5. $f(x) = \frac{x+1}{\sqrt{2x+3}} = \frac{x+1}{(2x+3)^{1/2}}$

$= \frac{(2x+3)^{1/2}(1) - (x+1)\left(\frac{1}{2}(2x+3)^{-1/2}(2)\right)}{2x+3}$

$= \frac{\sqrt{2x+3} - \frac{x+1}{\sqrt{2x+3}}}{2x+3} = \frac{2x+3-x-1}{(2x+3)^{3/2}} = \frac{x+2}{(2x+3)^{3/2}}$

6. $f(x) = 3(1+\sqrt{x})^7$

$f'(x) = 3 \cdot 7(1+x^{1/2})^6\left(\frac{1}{2}x^{-1/2}\right)$

$= \frac{21}{2}\frac{(1+\sqrt{x})^6}{\sqrt{x}}$

(20) III. Find the following limits.

1. $\underset{x \to -2^-}{\text{limit}} \frac{x}{x+2} = \frac{\ominus}{\ominus} = +\infty$

2. $\underset{x \to \infty}{\text{limit}} \frac{x^3}{2x^3+x} = \frac{1}{2+\frac{1}{x^2}} \longrightarrow \frac{1}{2}$

3. $\underset{x \to -\infty}{\text{limit}} \left(\frac{1}{x} - x\right) = +\infty$

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4. $\underset{x \to 1}{\text{limit}} \ \frac{x^2-1}{x-1}$ $= x+1 = 2$

(10) IV. Prove that $D(f(x) + g(x)) = Df(x) + Dg(x)$.

$$\frac{f(y)+g(y) - (f(x)+g(x))}{y-x} = \frac{f(y)-f(x)}{y-x} + \frac{g(y)-g(x)}{y-x}$$

$$D(f(x)+g(x)) = \lim_{y \to x} \frac{f(y)-f(x)}{y-x} + \lim_{y\to x} \frac{g(y)-g(x)}{y-x}$$

$$= f'(x) + g'(x)$$

(20) V. Find the equation of the lines tangent to each of the following curves at the indicated points.

1. $y = x^3 - 3x$ at $(2,2)$

$y' = 3x^2 - 3$

$m = f'(2) = 12 - 3 = 9$

$y - 2 = 9(x-2)$

$y - 2 = 9x - 18$

$y = 9x - 16$

2. $x^2 - y^2 = 5$ at $(3,-2)$

$2x - 2y\,Dy = 0$

$Dy = \frac{2x}{2y} = \frac{x}{y}$

$m = \frac{3}{-2}$

$y + 2 = -\frac{3}{2}(x-3)$

$y = -\frac{3}{2}x + \frac{9}{2} - \cancel{4}$

$\frac{5}{2}$

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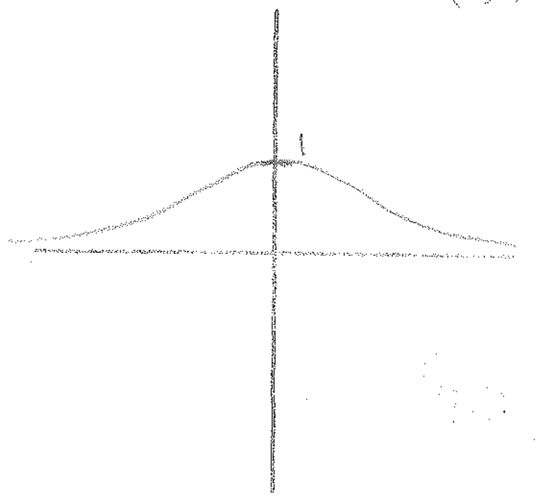
(10) VI. Sketch the graph of $y = \frac{1}{1+x^2}$, including horizontal and vertical asymptotes, if any.

(0,1)

$0 \le \frac{1}{1+x^2} \le 1$



$\lim_{x \to \pm\infty} \frac{1}{1+x^2} = 0$

$\boxed{y = 0}$

no vert

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