Math 131
Final Exam
Dec. 12, 1973

Name KEY

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23
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Show work for partial credit!

2a.

1. Find the indicated derivatives: (do not simplify!)

a. $f(x) = x^3 - 6x - x\sqrt{x}$ $\;(x^{3/2})$, $f'(x) = 3x^2 - 6 - \frac{3}{2}x^{+1/2}$

b. $f(x) = \sqrt{6x^2 - x}$, $f'(x) = \frac{1}{2}(6x^2 - x)^{-1/2}(12x - 1)$

c. $D\left[\frac{x}{x^2-3}\right] = \dfrac{(x^2-3)\cdot 1 - x(2x)}{(x^2-3)^2}$

d. $\frac{\partial}{\partial x}(xy^2 + 2x - 6y + 1) = y^2 + 2 + 0$

e. $f(x) = (x^2-3)(x^3+x+4)$, $f'(x) = (x^2-3)(3x^2+1) + (x^3+x+4)(2x)$

f. $\frac{\partial^2}{\partial x^2}(x^3y^2 + 3xy^3) = \frac{\partial}{\partial x}(3x^2y^2 + 3y^3) = 6xy^2 + 0$

g. $D\,(x^2-x-3)^7 = 7(x^2-x-3)^6(2x-1)$

7 1. h. $f(x) = (3x+5)^4$, $f''(x) = D(4(3x+5)^3(3))$

$= D\ 12(3x+5)^3 = 36(3x+5)^2 \cdot 3$

$= 108(3x+5)^2$

(5 2. Find the derivative of $f(x) = 2x^2 - x + 6$ using the definition of the derivative.

$$\lim_{h \to 0} \frac{2(x+h)^2 - (x+h) + 6 - (2x^2 - x + 6)}{h}$$

$$= \lim_{h \to 0} \frac{2\cancel{x^2} + 4xh + 2h^2 - \cancel{x} - h + \cancel{6} - 2\cancel{x^2} + \cancel{x} - \cancel{6}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}[4x + 2h - 1]}{\cancel{h}} = 4x - 1$$

0 3. For which values of x are each of the following functions discontinuous?

a. $f(x) = \frac{x^2-1}{x-1}$ $\qquad X = 1$

b. $f(x) = \frac{x^2}{x^2+1}$ $\qquad$ none

c. $h(x) = \begin{cases} x^2, & x \geq 1 \\ x, & x < 1 \end{cases}$ $\qquad$ none

$$\lim_{x \to 1} x^3 = 1$$

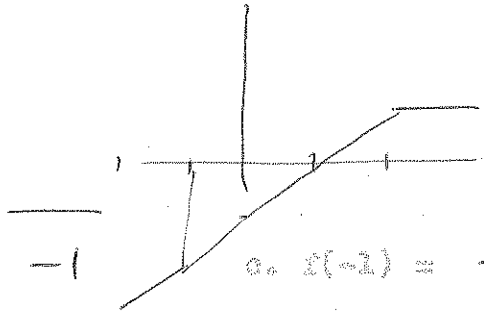
$$\parallel$$

$$\lim x = 1$$

$\overline{2}$

4 4. Suppose

$$f(x) = \begin{cases} 1, & x > 2 \\ x - 1, & -2 \le x \le 2 \\ -1, & x < -2 \end{cases}$$



Find each of the following:

a. $f(3) =$ $+1$ b. $f(0) =$ -1 c. $f(-1) =$ -2

d. $\lim_{x \to 1} f(x) =$ 0 e. $\lim_{x \to \infty} f(x) =$ 1

f. $\lim_{x \to -1^-} f(x) =$ -2 g. $\lim_{x \to -2^+} f(x) =$ -3

2ea

0 5. Find the equation of the straight line tangent to the curve $y = 3x^4 - 5x^2 + 4$ at the point with x coordinate 1 on the curve.

$y' = 12x^2 - 10x$ $y = 3(1) - 5(1) = -2$

at 1 $m = 12 - 10 = 2$ $+4 = 2$

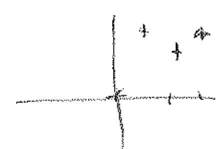
$(1, -2)$
$m = 2$

$$\frac{y \mp 2}{x - 1} = 2$$

$$y \mp 2 = 2x - 2$$

$$y = 2x \cancel{+4}$$

.5 6. Find the absolute maximum and absolute minimum value for $f(x) = 2x^3 - 9x^2 + 12x$ on $[0, 3]$.



$f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2)$

$6(x-2)(x-1)$

cp $x = 1$, $x = 2$

$x = 1$ $f(1) = 2 - 9 + 12 = \underline{5}$ max $f(3) = 19$

$f(2) = \underset{16}{2 \cdot 8} - \overset{36}{9 \cdot 4} + 24 = \underline{4}$ min $f(0) = 0$

$f(0) = \underline{0}$

$f(3) = 2 \cdot 27 - 9 \cdot 9 + 12 \cdot 3$

$54 - 81 + 36$

$= 90 - 81 = \underline{9}$

0 7. Find the following limits:

a. $\lim\limits_{x \to 1^+} \frac{2}{x-1} =$ $\frac{1}{+}$ $+\infty$

b. $\lim\limits_{x \to \infty} \frac{2x^2 - 2}{x^2 - x + 1} =$ $\lim\limits_{x \to \infty} \frac{2 - 2/x^2}{1 - 1/x + 1/x^2} = 2$

c. $\lim\limits_{x \to 2} \frac{x^3 - 1}{x^3 + 1} =$ $\frac{8-1}{8+1} = \frac{7}{9}$

d. $\lim\limits_{x \to 1^+} \frac{x-1}{\sqrt{x} - 1} =$ $\lim\limits_{x \to 1^+} \frac{\cancel{(x-1)}(\sqrt{x}+1)}{\cancel{x-1}} = 2$

10 8. A rock is thrown up into the air from a cliff. Two seconds later it reaches a maximum height of 300 feet and begins to fall. How high is the cliff?

$s(2) = 300$

$v(2) = 0$

$a(t) = -32$

$v(t) = -32t + c$

$0 = -64 + c$

$c = 64$

$v(t) = -32t + 64$

$s(t) = -16t^2 + 64t + C$

$300 = -16(4) + 64(2) + C$

$C = 300 + 64 - 128$

$= 300 - 64 = 236$

$s(t) = -16t^2 + 64t + 236$

$s(0) = 236$

0 9. A particle moves along a line according to the function $s(t) = 4t^2 - 16t + 30$, where distance is in feet and time in seconds.

a. What is the velocity when $t = 2$?

$v(t) = 8t - 16$

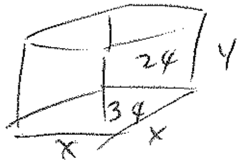
$v(2) = 16 - 16 = 0$

b. At what point(s) does the particle turn around?

$s(2) = 4 \cdot 4 - 16 \cdot 2 + 30$

$= -16 + 30 = \underline{14}$

5 10. A box with square bottom is to have volume 12 cu. in. If the material for the sides costs 2 cents per sq. in. and for the top and bottom costs 3 cents per sq. in., find the dimensions of the box which has minimum cost.



$x^2 y = 12 \qquad y = \frac{12}{x^2}$

$C(x) = 2x^2(3) + 4xy(2)$

$= 6x^2 + 8xy$

$= 6x^2 + 8x \cdot \frac{12}{x^2}$

$= 6x^2 + \frac{96}{x}$

$C'(x) = 12x - 96x^{-2} = 0$

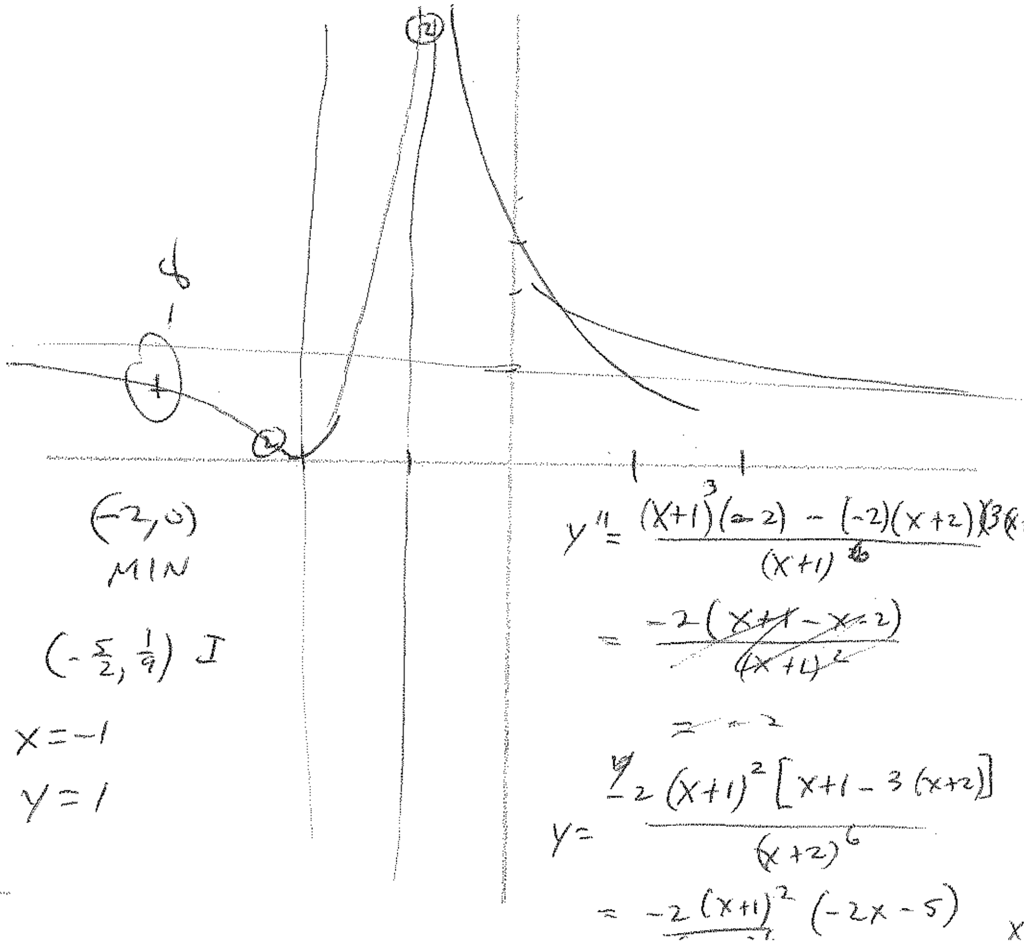
5

$2 \times 2 \times 3$

$\frac{x^3 - 8}{x^2} = 0$

$x = 2 \quad y = \frac{12}{4} = 3$

.5 11. Carefully sketch the graph of $y = \frac{(x+2)^2}{(x+1)^2}$. Draw all asymptotes, and find their equations. Plot and find the coordinates of local maxima, minima, and points of inflection.



always pos

$\lim \frac{(x+2)^2}{(x+1)^2} = 1$

$y' = \frac{(x+1)^2(2(x+2)) - (x+2)^2\, 2(x+1)}{(x+1)^4} = \frac{2(x+1)(x+2)\left(x+1-(x+2)\right)}{(x+1)^4}$

$= \frac{2(x+2)(\cancel{2}x-1)}{(x+1)^3}$

$= \frac{-2(x+2)}{(x+1)^3}$

CP
$x = -2$
$x = -1$

$(-2, 0)$
MIN

$(-\frac{5}{2}, \frac{1}{9})$ I

$x = -1$
$y = 1$

$y'' = \frac{(x+1)^3(-2) - (-2)(x+2)3(x+1)^2}{(x+1)^6}$

$= \frac{-2(\cancel{x+1-x-2})}{\cancel{(x+1)^2}}$

$= -2$

$y = \frac{-2(x+1)^2[x+1-3(x+2)]}{(x+2)^6}$

$= \frac{-2(x+1)^2(-2x-5)}{\ldots}$

$x = -\frac{5}{2}$ CP $\quad y = \frac{(1/2)^2}{(3/2)^2} = \frac{1}{9}$