Math 151
Test II
Nov. 2, 1973

Name KEY

Show work for full credit!

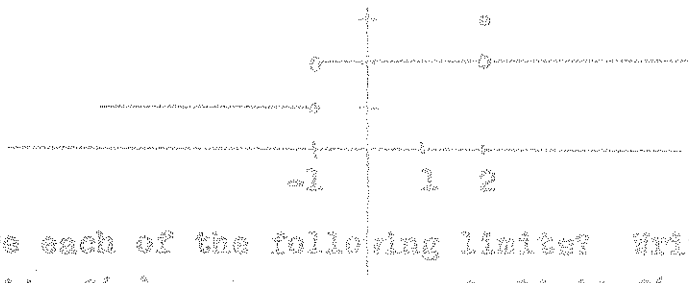
(10) 1. If $f(x) = x^3 - 3$, and $g(x) = 2x + 1$, find:

a. $g(f(x)) =$ $g(x^3 - 3) = 2(x^3 - 3) + 1 = 2x^3 - 5$

b. $g(g(2)) =$ $g(2 \cdot 2 + 1) = g(5) = 2(5) + 1 = 11$

c. $f(\sqrt{x}) =$ $(\sqrt{x})^3 - 3 = x^{3/2} - 3$

(10) 2. Suppose the function f has the following graph.



What are each of the following limits? Write DNE if it does not exist.

a. $\lim_{x \to -1^-} f(x) = 1$

b. $\lim_{x \to -1^+} f(x) = 2$

c. $\lim_{x \to -1} f(x) =$ DNE

d. $\lim_{x \to 2^-} f(x) = 2$

e. $\lim_{x \to 2^+} f(x) = 2$

f. $\lim_{x \to 2} f(x) = 2$

(10) 3. Evaluate the following limit using only one limit theorem at a time:

$$\lim_{x \to 4} \frac{2x}{x^2 + 3} = \frac{\lim_{x \to 4} 2x}{\lim_{x \to 4} (x^2 + 3)} = \frac{2 \lim_{x \to 4} x}{\lim_{x \to 4} x^2 + \lim_{x \to 4} 3}$$

$$= \frac{2 \lim_{x \to 4} x}{\lim_{x \to 4} x \cdot \lim_{x \to 4} x + \lim_{x \to 4} 3} = \frac{2 \lim_{x \to 4} x}{(\lim_{x \to 4} x)(\lim_{x \to 4} x) + \lim_{x \to 4} 3}$$

$$= \frac{2 \cdot 4}{4 \cdot 4 + 3} = \frac{8}{16 + 3} = \frac{8}{19}$$

(20) 4. Find each of the following limits:

a. $\lim_{x \to 2} (x^3 + 3x) =$ $2^3 + 3(2) = 8 + 6 = 14$

b. $\lim_{x \to 2} \frac{x}{x+1} =$ $\frac{2}{3}$

c. $\lim_{x \to -4} \frac{x^2 - 16}{x+4} =$ $\lim (x+4)(x-4) = -8$

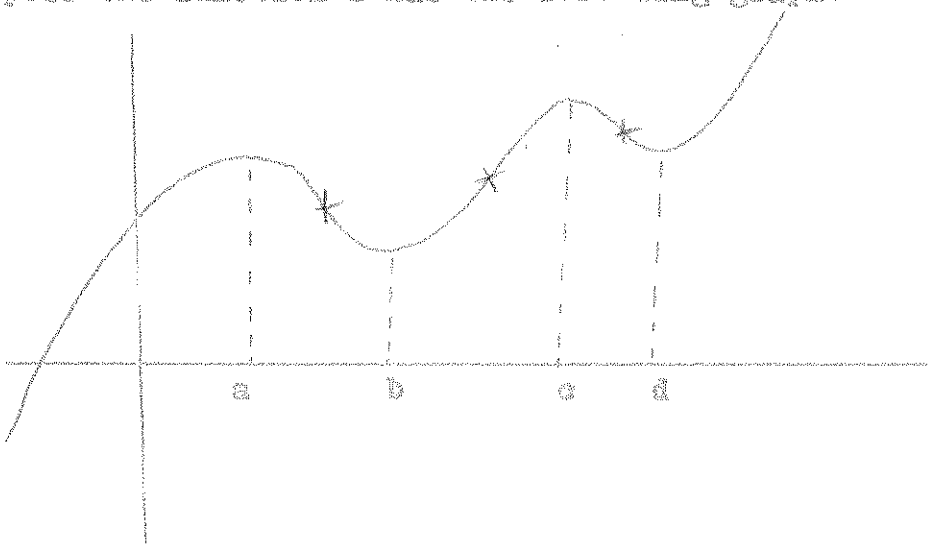
d. $\lim_{x \to \infty} \frac{x^2 - x + 1}{3x^2 + 5} =$ $\lim_{x \to \infty} \frac{1 - \frac{1}{x} + \frac{1}{x^2}}{3 + 5/x^2} = \frac{1}{3}$

e. $\lim_{x \to 3^-} \frac{4}{x-3} =$ $-\infty$

3 ans

f. $\lim_{x \to 0^+} \frac{5}{x} =$ $+\infty$

(15) 5. Suppose the function f has the following graph:



a. Mark all points of inflection with an x.

b. Indicate for each of the following intervals whether f is increasing on that interval.

i. $[a,b]$ Yes( ) No(X)

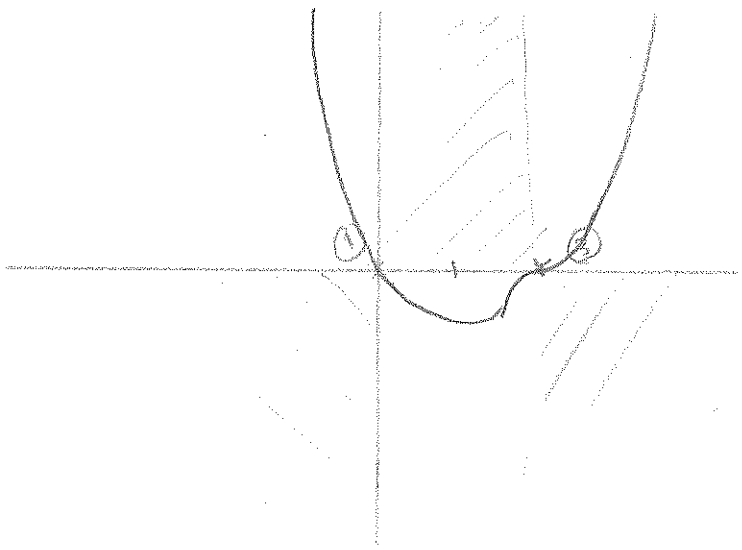
ii. $[b,c]$ Yes(X) No( )

iii. $[a,c]$ Yes( ) No(X)

iv $[b,\infty)$ Yes( ) No(X)

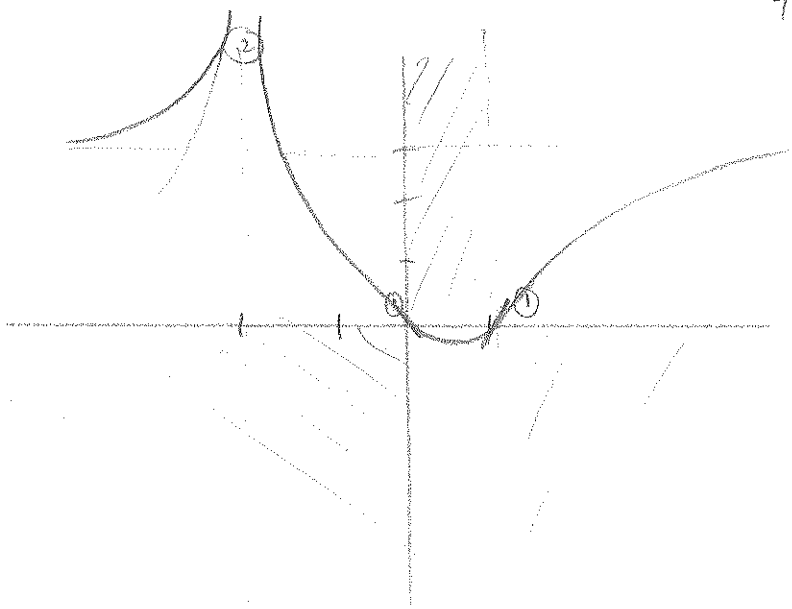
(35) 6. Sketch the graph of each of the following. Draw and give the equation of all asymptotes.

a. $f(x) = x(x-2)^3$



no asymp.

b. $g(x) = \frac{3x(x-1)}{(x+2)^2}$



vert $x = -2$

$$\lim_{x\to\infty} \frac{3x^2 - 3x}{x^2+4x+4}$$

$$= \lim_{x\to\infty} \frac{3 - \frac{3}{x}}{1 + \frac{4}{x} + \frac{4}{x^2}} = 3$$

hori $y = 3$

$3x(x-1) = 3$

$3x^2 - 3x = 3$

$x^2 - x = 1$

$x^2 - x + 1 = 0$

$x \quad \sqrt{1-4}$