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Test III
Nov. 20, 1973 Name KET

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5 1. Complete the following definition: The <u>derivative</u> of a function f is the function f' given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

10 2. Find the derivative of the function $f(x) = x^2-5x-3$ using the definition. Check using other methods.

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 5(x+h) - 3 - (x^2 - 5x - 3)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 5x - 5h - 3 - x^2 + 5x + 3}{h}$$

$$= \frac{2xh + h^2 - 5h}{h} = 2x - 5 + h$$

$$\lim_{h \to 0} (2x - 5 + h) = 2x - 5.$$

10 3. Show that $D[f(x)+g(x)] = Df(x)+Dg(x)$ ($= f'(x)+g'(x)$.)

$$D[f(x)+g(x)] = \lim_{h \to 0} \frac{f(x+h) + g(x+h) - (f(x) + g(x))}{h}$$

$$= \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right)$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= f'(x) + g'(x)$$

25

. Find the following derivatives:

a. $f(x) = x^3 - 6x - 7$, $f'(x) = $ $3x^2 - 6$

b. $g(x) = x^7 - \frac{6}{x^2} + \sqrt{x}$, $g'(x) = $ $7x^6 - 6(-2)x^{-3} + \frac{1}{2}x^{-1/2}$

$= 7x^6 + 12x^{-3} + \frac{1}{2}x^{-1/2}$

$= 7x^6 + \frac{12}{x^3} + \frac{1}{2\sqrt{x}}$

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c. $f(x) = 2(3x - 7)^6$, $f''(x) = $

$f' = 12(3x-7)^5(3) = 36(3x-7)^5$

$f'' = 180(3x-7)^4(3) = 540(3x-7)^4$

d. $D\left[\frac{x^2}{x^2+1}\right] = \frac{(x^2+1)(2x) - x^2(2x)}{x^2+1} = \frac{2x[x^2+1-x^2]}{x^2+1}$

$= \frac{2x}{x^2+1}$

e. $D\sqrt{x^2-2x+1} = \frac{1}{2}(x^2-2x+1)^{-1/2}(2x-2)$

$= \frac{x-1}{\sqrt{x^2-2x+1}}$

f. $D\left[(2x-5)(x^2+6x-1)\right] = (2x-5)(2x+6) + (x^2+6x-1)2$

20

5. On what intervals is the function $f(x) = x^3(x-4) = x^4 - 4x^3$ strictly increasing? dec?
Find the coordinates of all local maxima and minima.

$$f' = 4x^3 - 12x^2 = 4x^2[x-3]$$

I $x \geq 3$ $\qquad f'' = 12x^2 - 24x$

$12x[x-2]$

CP

$x = 0 \qquad x = 3$

$f'' = 0 \qquad f'' > 0$

X $\qquad$ MIN

$(3, -27)$

5.

15

10 6. On what intervals is the graph of the function $f(x) = x^4(x+5)$ concave up?
Find the coordinates of all points of inflection.

$f = x^5 + 5x^4$

$f' = 5x^4 + 20x^3$

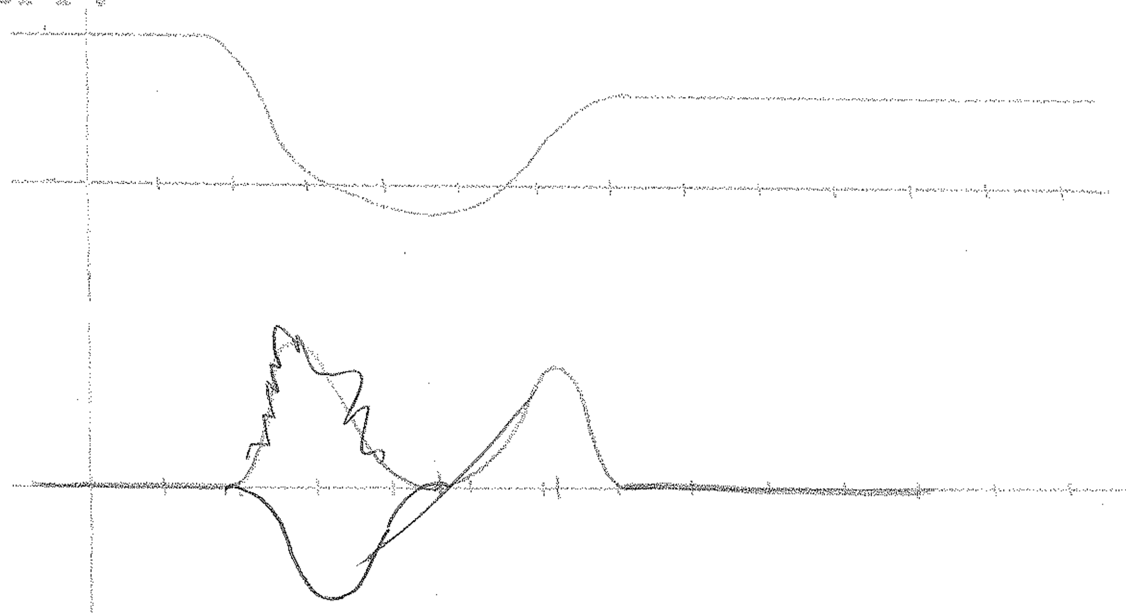
$f'' = 20x^3 + 60x^2 \qquad x > -3$

$20x^2[x+3]$

$0, -3$

$(-3, 162)$

10 7. Below is the graph of the function f. Sketch from this the graph of f'.



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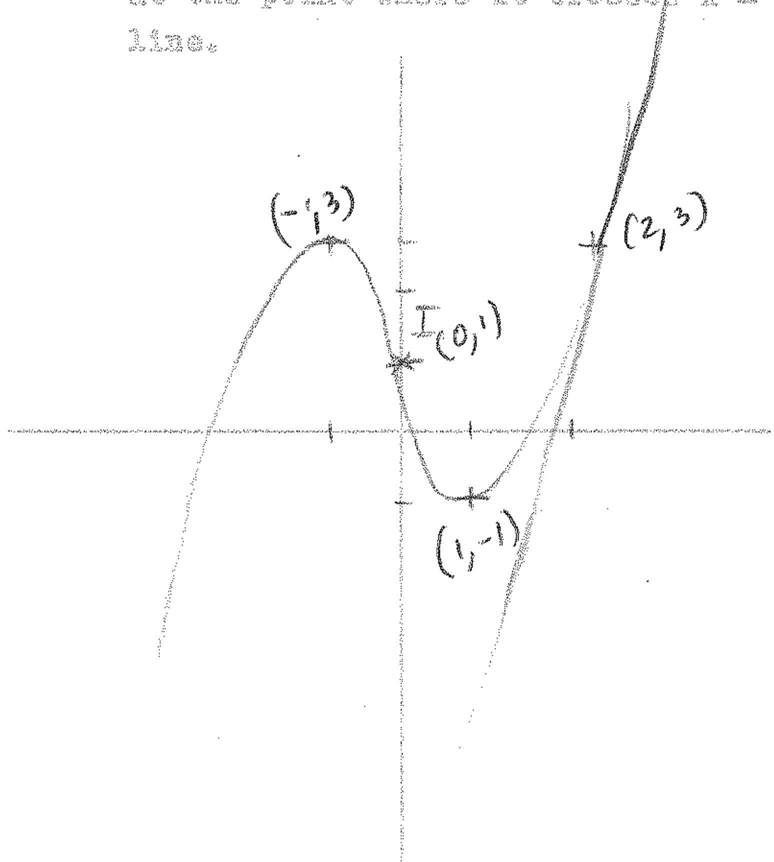
4. $y = x^3 - 3x + 1$

a. Carefully sketch the graph of this curve. Plot and label all local maxima and minima, and points of inflection (give coordinates

b. On the same graph sketch the straight line tangent to the curve at the point where it crosses $x = 2$. Find the equation of this line.



$y' = 3x^2 - 3$

$y'' = 6x$

$(0,1)$ PI

CP (x^2-1) $x = \pm 1$

1 $y'' = 6 > 0$

$(1,-1)$ min

-1 $y'' = -6 < 0$

$(-1,3)$ max

$1,\ 1 - 3 + 1 = -1$

$-1,\ -1 + 3 + 1 = 3$

$2 \quad 8 - 6 + 1 = 3$

$y' = 3(4) - 3 = 9$

$y - 3 = 9(x - 2)$

$y = 9x - 18 + 3$

$= 9x - 15$

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