Math 131
Final Exam
Dec. 10, 1974

Name KEY

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(45) I. Short answer, no partial credit: 17 mi

3 ea

1. $\frac{d}{dx}(x^3 - \frac{2}{x} + 3) = 3x^2 + 2x^{-2}$ ($\frac{2}{x}$: $x^{-1}$)

2. $\lim_{x\to\infty} \frac{3}{x-1} = 0$

3. $\lim_{x\to -3^-} \frac{-3}{x+3} = +\infty$

4. $\lim_{x\to 1} \frac{x^2-1}{x-1} = x+1$ $\boxed{2}$

5. If $y = 2x^6$, then $\frac{d^3y}{dx^3} = 12x^5 \quad 60x^4 \quad 360x^3$

6. If $f(x) = \frac{x^2-9}{x-3}$ ($x+3$) for $x \neq 3$, then in order for $f$ to be continuous at $x = 3$, we must have $f(3) = 6$

7. If $y = \frac{\sqrt{x}}{x^2}$ ($x^{-3/2}$), then $\frac{dy}{dx} = -\frac{3}{2}x^{-5/2}$ or $\frac{x^2\frac{1}{2\sqrt{x}} - \sqrt{x}(+2x^{-1})}{x^4}$

$\frac{1}{2x^2\sqrt{x}} - 2\frac{\sqrt{x}}{x^3}$

$\frac{1}{2x^2\sqrt{x}} + \frac{\sqrt{x}}{x^7}$

8. If $u = x^2+1$ and $y = u^3+1$, write $y$ in terms of $x$,

$y = (x^2+1)^3+1$ $\quad x^6 + 3x^4 + 3x^2 + 1 + 1$

9. If $f'''(x) \geq 0$ for all $x$, what can be said about the graph of $f''$?

$f''$ incr

10. $\frac{d}{dx}(1+\sin 2x) = 2\cos 2x$

11. $\int x^3 - 6x \, dx = \frac{x^4}{4} - \frac{6x^2}{2} + C = \frac{x^4}{4} - 3x^2 + C$

12. If $\frac{dy}{dx} = x^3+x$, then $y = \frac{x^4}{4} + \frac{x^2}{2} + C$

13. $\int \sin 3x \, dx = -\frac{\cos 3x}{3} + C$

14. $\cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$

15. The slope of the line tangent to the curve $y = \sqrt{x}$ at (4,2) is

$\frac{1}{2\sqrt{x}}$ $\quad \frac{1}{4}$

45

NO ARITIT.

II. Find the derivatives below:

Ten 1. $y = \sqrt{x^3+1}$ , $\frac{dy}{dx} =$ $\frac{1}{2}(x^3+1)^{-1/2}\, 3x^2 = \frac{3x^2}{2\sqrt{x^3+1}}$

2. $y = (x^2+1)(2x-1)$, $\frac{dy}{dx} = (x^2+1)(2) + (2x-1)(2x)$

$= 2x^2+2 + 4x^2 - 2x$

$= 6x^2 - 2x + 2$

3. $f(x) = \frac{x^2-1}{x^2+1}$ , $\quad$ * or $(x^2-1)(x^2+1)^{-1} \rightarrow (x^2-1)(-1)(x^2+1)^{-2}(2x) + (x^2+1)^{-1}(2x)$

$f'(x) = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2} = \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}$

$= \frac{4x}{(x^2+1)^2}$

4. $x^2+3xy+y^2 = 7$.

$\frac{dy}{dx} =$ $\quad 2x + 3x\frac{dy}{dx} + 3y + 2y\frac{dy}{dx} = 0$

$(3x+2y)\frac{dy}{dx} = -2x-3y$

$\frac{dy}{dx} = \frac{-2x-3y}{3x+2y} = -\frac{2x+3y}{3x+2y}$

5. $z = x^3 - xy^2$,

$\frac{\partial z}{\partial x} = 3x^2 - y^2$

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II. cont.

(7) 6. $y = (1+ \sin 2x)^3$, $\frac{dy}{dx} =$ $3(1+\sin 2x)^2 \cos 2x \cdot 2$

$= 6 \cos 2x (1+\sin 2x)^2$

(10) III. The equation of the straight line tangent to the ellipse $x^2+4y^2 = 4$ at the point $(1, -\frac{\sqrt{3}}{2})$ is

$2x + 8y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{4y}$

$x=1$
$y=-\frac{\sqrt{3}}{2}$ $= \frac{-1}{4(-\frac{\sqrt{3}}{2})} = \frac{1}{2\sqrt{3}}$

$\frac{y+\frac{\sqrt{3}}{2}}{x-1} = \frac{1}{2\sqrt{3}}$

$y+\frac{\sqrt{3}}{2} = \frac{x}{2\sqrt{3}} - \frac{1}{2\sqrt{3}}$

$y = \frac{x}{2\sqrt{3}} - \frac{\sqrt{3}}{6} - \frac{\sqrt{3}}{2}$

$y = \frac{x}{2\sqrt{3}} - \frac{2\sqrt{3}}{3}$

9, 3

(13) IV. Using only the definition, find the derivative of $f(x) = x^2+5x+2$.

$f(x+\Delta x) = (x+\Delta x)^2 + 5(x+\Delta x) + 2$

$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{(x+\Delta x)^2 + 5(x+\Delta x) + 2 - (x^2+5x+2)}{\Delta x}$

$= \frac{\cancel{x^2} + 2x\Delta x + (\Delta x)^2 + \cancel{5x} + 5\Delta x + \cancel{2} - \cancel{x^2} - \cancel{5x} - \cancel{2}}{\Delta x}$

$= \frac{\cancel{\Delta x}(2x + \Delta x + 5)}{\cancel{\Delta x}}$

$f'(x) = \lim_{\Delta x \to 0} (2x + \Delta x + 5) = 2x+5$

V. Without using calculus, sketch a graph of $y = \frac{(x+2)^2}{(x+1)^2}$. $\geq 0$

Sketch and find the equations of all asymptotes.

Zeros $x=-2$

pole $x=-1$

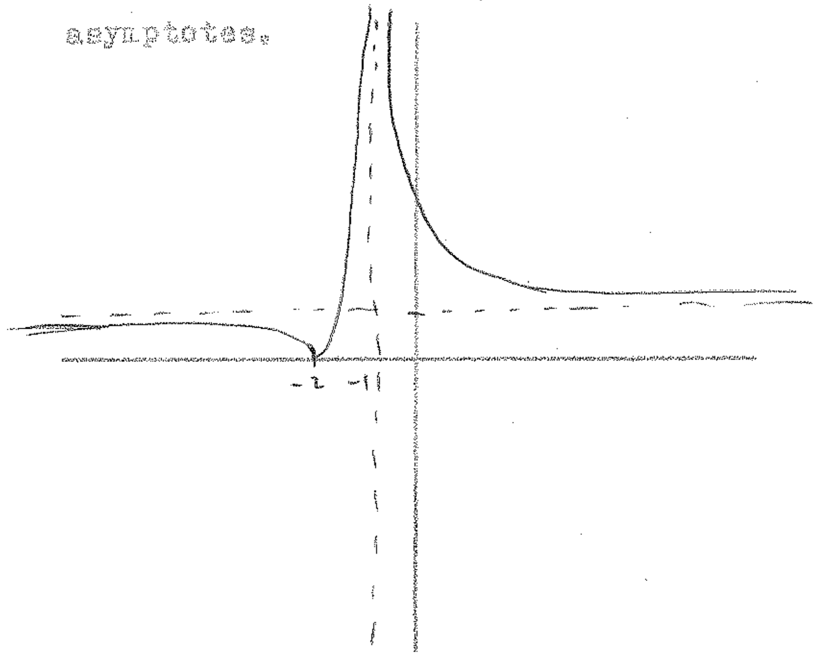
$x \to \infty,\ y \to 1$

asyp

$x=-1$ 2er

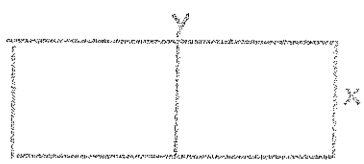
$y=1$

15




VI.

(15) 1. A fence is to be built to enclose an area of 15,000 sq. ft., and there is to be a dividing fence as shown. What should be the dimensions so that the total length of fence used is a minimum?




$15{,}000 = xy \qquad y = \frac{15000}{x}$

$L = 3x + 2y$

$L = 3x + 30{,}000x^{-1}$

$\frac{dL}{dx} = 3 - 30{,}000x^{-2} = 3 - \frac{30{,}000}{x^2} = 0$

$\cancel{3}x^2 = \cancel{3}0{,}000$

$\boxed{\begin{array}{l} x = 100 \\ y = 150 \end{array}}$

OR

$x = \frac{15{,}000}{y}$

$L = \frac{45{,}000}{y} + 2y$

$\frac{dL}{dy} = -45{,}000y^{-2} + 2 = 0$

$y^2 = \frac{45{,}000}{2}$

$y^2 = 22{,}500$

$y = 150$

$x = 100$

(30) VI. 2. $f(x) = x^4-4x^3+6$

$f'(x) = 4x^3-12x^2 = 4x^2(x-3)$ $\quad >0 \;\; x>3 \quad <0 \;\; x<3$

$f''(x) = 12x^2-24x = 12x(x-2)$

| | | | |
|---|---|---|---|
| $x<0$ | $-$ | $-$ | $+$ |
| $0<x<2$ | $+$ | $-$ | $-$ |
| $x>2$ | $+$ | $+$ | $+$ |

a. For what x is f increasing?

$x>3$

b. For what x is f decreasing?

$x<3$

c. For what x is the graph of f concave up?

$x<0, \; x>2$

d. For what x is the graph of f concave down?

$0<x<2$

e. What are the coordinates of the local maxima and minima?

C.P. $x=0$ neither

$x=3$ $\quad x>$ MIN ~~max~~ $\quad$ ~~MAX~~ MIN $(3,-21)$

non MAX.

$\frac{27}{4}$

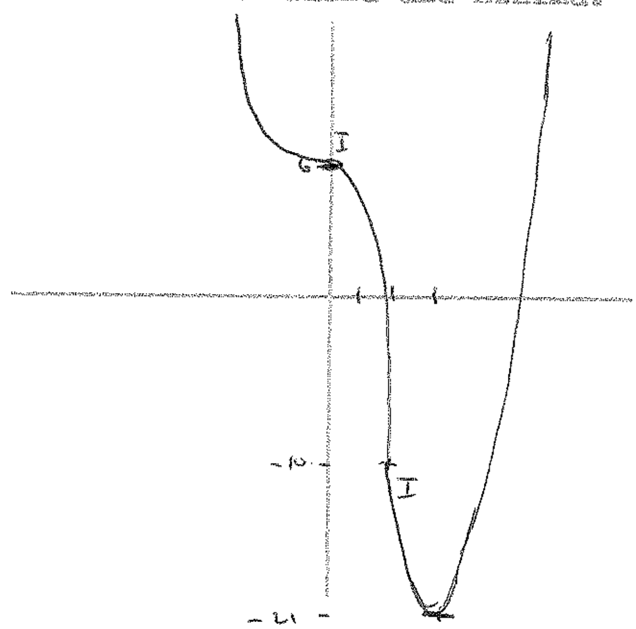
81
6
87
−108

f. What are the coordinates of the points of inflection?

$(0,6)$ $(2,-10)$

16
−32
+6

g. Sketch the graph of f. Plot and label the points of inflection and maxima and minima.

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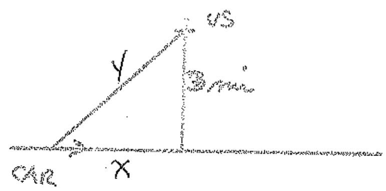
(15) VI. 3. A car is travelling towards us at 30 mi./hr. along a straight road which is 3 miles away from us at the closest point. When the car is 5 miles away, at what rate is the distance between us and the car changing?




$$\frac{dx}{dt} = -30$$

$$y^2 = 9 + x^2$$

$$\cancel{2}y\frac{dy}{dt} = \cancel{2}x\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$$

$y = 5$

$x = 4$ $\quad \dfrac{dy}{dt} = \dfrac{4}{5}(-30) = \boxed{-24}$

OR

$$y = \sqrt{9 + x^2}$$

$$\frac{dy}{dt} = \frac{1}{\cancel{2}}\left(\sqrt{9+x^2}\right)^{-1/2} \cancel{2}x\frac{dx}{dt}$$

$$= \frac{x}{\sqrt{9+x^2}}\frac{dx}{dt}$$

$x = 4$ $\quad = \dfrac{4}{5}(-30) = -24$

(15) 4. A projectile is fired directly upward with an initial velocity of ~~1200~~ 800 ft/sec. If the acceleration due to gravity is 32 ft/sec/sec find how high the projectile goes before it falls back to earth.

$a = -32$

$V = -32t + C$

$t = 0 \quad V = 800$

$V = -32t + 800$

$S = -16t^2 + 800t + c$

$t = 0 \quad s = 0$

$S = -16t^2 + 800t$

$$\begin{array}{r} 625 \\ \times 16 \\ \hline 3750 \\ 625 \\ \hline 10000 \end{array}$$

$V = 0$

$-32t + 800 = 0$

$t = \dfrac{800}{32} = 25$

$S = -16(25)^2 + 800(25)$

$= -10{,}000 + 20{,}000 = 10{,}000$ ft.