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304
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Math 131
Test I
October 11, 1974

Name KEY

(3 ea) I. Short answer:

1. The slope of the straight line $x + 3y = 6$ is $-\frac{1}{3}$

$3y = -x + 6$

2. The equation of the vertical line through the point $(-2,7)$ is $y = -2$

3. The equation of the straight line through the origin $(0,0)$ parallel to the line $y = 3x + 5$ is $y = 3x$

4. The slope of the line through $(2,12)$ and $(-3,9)$ is

$\frac{12-9}{2-(-3)} = \frac{3}{5}$

5. The equation of the circle with center $(1,-2)$ and radius 9 is

$(x-1)^2 + (y+2)^2 = 81$ $x^2 - 2x + y^2 + 4y = 76$

$x^2 - 2x + 1 + y^2 + 4y + 4 = 81$

6. The distance between the points with coordinates $(1,5)$ and $(4,1)$ is

$\sqrt{(4-1)^2 + (5-1)^2} = \sqrt{3^2 + 4^2} = 5$

7. If a tank contains 50,000 gallons when the drain is opened, and 3 hours later 35,000 remain; what is the average rate of change of the quantity in the tank with respect to time? (Include units of measurement)

$\frac{35{,}000 - 50{,}000}{3} = \frac{-15{,}000}{3}$

$= -5{,}000$ gal/hr

8, 9. If $f(x) = 5x^2 - x$, then $f(2) = 5 \cdot 4 - 2 = 18$

and $f(x+3) = 5(x+3)^2 - (x+3) = 5(x^2 + 6x + 9) - (x+3)$

$= 5x^2 + 30x + 45 - x - 3$

$= 5x^2 + 29x + 42$

10. Give two different notations for the derivative:

$\frac{dy}{dx}$, $f'(x)$

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II. On the graph paper sketch carefully the graph of $y = x(x-1)^3(x+2)^2$.

$x = -2 \quad x = 0 \quad x = 1$

$x < -2$	$-$	$-$	$+$	$+$
$-2 < x < 0$	$-$	$-$	$+$	$+$
$0 < x < 1$	$+$	$-$	$+$	$-$
$x > 1$	$+$	$+$	$+$	$+$

III. Do EITHER 1 or 2. #1 is worth 15 points, #2 is worth 20.

Using the definition of the derivative, find the derivative of

1. $f(x) = x^2+2x-7$

OR 2. $f(x) = (2x-1)^{3/2}$

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1. $$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{(x+\Delta x)^2+2(x+\Delta x)-7-(x^2+2x-7)}{\Delta x}$$

$$= \frac{\cancel{x^2}+2x\Delta x+(\Delta x)^2+\cancel{2x}+2\Delta x\cancel{-7}\cancel{-x^2}\cancel{-2x}\cancel{+7}}{\Delta x}$$

$$= \frac{2x\Delta x+(\Delta x)^2+2\Delta x}{\Delta x} = \frac{\cancel{\Delta x}(2x+\Delta x+2)}{\cancel{\Delta x}}$$

$$f'(x) = \lim_{\Delta x \to 0}(2x+\Delta x+2) = 2x+2$$

2. $$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{(2(x+\Delta x)-1)^{3/2}-(2x-1)^{3/2}}{\Delta x} = \frac{\sqrt{(2(x+\Delta x)-1)^3}-\sqrt{(2x-1)^3}}{\Delta x}$$

$$= \frac{\sqrt{(2(x+\Delta x)-1)^3}-\sqrt{(2x-1)^3}}{\Delta x}\cdot\frac{\sqrt{(2(x+\Delta x)-1)^3}+\sqrt{(2x-1)^3}}{\sqrt{(2(x+\Delta x)-1)^3}+\sqrt{(2x-1)^3}}$$

$$= \frac{(2(x+\Delta x)-1)^3-(2x-1)^3}{\Delta x(\sqrt{\ }+\sqrt{\ })} = \frac{8(x+\Delta x)^3-3\cdot4(x+\Delta x)^2+3\cdot2(x+\Delta x)-1-(8x^3-3\cdot4x^2+3\cdot2(x)-1)}{\Delta x(\sqrt{\ }+\sqrt{\ })}$$

$$= \frac{8(x^3+3x^2\Delta x+3x(\Delta x)^2+(\Delta x)^3)-12(x^2+2x\Delta x+(\Delta x)^2)+6(x+\Delta x)-1-8x^3+12x^2-6x+1}{\Delta x(\sqrt{\ }+\sqrt{\ })}$$

$$= \frac{\cancel{8x^3}+24x^2\Delta x+24x(\Delta x)^2+8(\Delta x)^3\cancel{-12x^2}-24x\Delta x-12(\Delta x)^2+\cancel{6x}+6\Delta x\cancel{-1}\cancel{-8x^3}\cancel{+12x^2}\cancel{-6x}\cancel{+1}}{\Delta x(\sqrt{\ }+\sqrt{\ })}$$

$$= \frac{24x^2\Delta x+24x(\Delta x)^2+8(\Delta x)^3-24x\Delta x-12(\Delta x)^2+6\Delta x}{\Delta x(\sqrt{\ }+\sqrt{\ })}$$

$$= \frac{24x^2+24x(\Delta x)+8(\Delta x)^2-24x-12(\Delta x)+6}{\cancel{\Delta x}(\sqrt{\ }+\sqrt{\ })}$$

$$\longrightarrow 24x^2-24x+6 \Big/ \sqrt{(2x-1)^3} = \frac{6(4x^2-4x+1)}{\sqrt{\ }} = 3(2x-1)^{1/2}$$

IV. In the remaining problems you may use the fact that if $f(x) = ax^2+bx+c$, the $f'(x) = 2ax+b$, for any a, b, c.

15 1. The distance in feet that a free-falling object has fallen in t seconds is given by $s = 16t^2$. What is the velocity of this object when $t = 2$? What is the velocity when it has fallen 100 feet? (units)

$v = s' = 32t$

$t = 2 \quad v = 64$ ft/sec

$s = 100$

$16t^2 = 100$

$t^2 = 25$

$t = \frac{5}{2}$

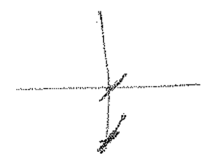
$t = 5/2$

$v = 32 \cdot \frac{5}{2} = \cancel{160}\ 80$ ft/sec

10 2. Find the slope of the straight line tangent to the curve $y = x^2+x-5$ at the point $(1, -3)$.

$y' = 2x+1$

$m = 2(1)+1 = 3$



15 3. $y = 2x^2-4x \qquad 2x(x-2)$

a. On separate graph paper sketch the graph using the method in class. Include the tangents at the zeros. $y' = 4x-4$

b. Find the equation of the horizontal straight line which tangent th this curve.

$4x-4=0$

$x = 1$

$y = 2-4 = -2$

$\boxed{y=-2}$

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II