Math 131
Test II
Nov. 4, 1974

Name KEY

I. Short answer:

3 ea 1. $\frac{d}{dx}(x^2-2x) =$ $2x-2$

2. $y = 3\sqrt{x}$, $\frac{dy}{dx} =$ $\frac{3}{2}x^{-1/2} = \frac{3}{2\sqrt{x}}$

3. $y = 5^2$, $\frac{dy}{dx} =$ 0

4. $f(x) = -\frac{3}{x^3} + x^2$ (x^{-3}), $f'(x) =$ $-9x^{-4}+2x$

5. $\lim\limits_{x\to 2} (x^2+3) =$ 7

6. $\lim\limits_{x\to 3+} \frac{x^2}{x-3} =$ $+\infty$

7. $\lim\limits_{x\to 3} \frac{x^2-9}{x-3} =$ 6

8. $\lim\limits_{x\to\infty} \frac{x^2-2x+1}{3x^2-x} =$ $\frac{1}{3}$

9. For what values of x is $f(x) = \frac{x^2}{(x-1)^2}$ discontinuous?

$x=1$

10. A. The limit of a product is the product of the limits.
B. The derivative of a product is the product of the derivatives.

Both are true ( ) Both are false ( )
A is true and B is false (X) B is true and A is false ( )

5 II. Use differentials to approximate $\sqrt[3]{8.01}$.

$f(x) = \sqrt[3]{x} = x^{1/3}$ $\qquad dx = .01$

$f'(x) = \frac{1}{3}x^{-2/3}$ $\qquad x = 8$

$dy = \frac{1}{3}x^{-2/3}dx$

$= \frac{1}{3\cdot 8^{2/3}}(.01) = \frac{1}{12}(.01) = \frac{1}{1200} = .0008\overline{3}$

$\sqrt[3]{8.01} \approx 2 + \frac{1}{1200}$

$1200\overline{)1.00000}$ $\quad .0008$
$\quad 9600$
$\quad 4000$

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III. Find the derivatives of each of the following. Use proper notation. Do not simplify algebra.

1. $y = (2x-5)^5$ $\qquad \frac{dy}{dx} = 5(2x-5)^4(2)$

2. $f(x) = (x^2-x+1)(x^3+3)$

$$f'(x) = (x^2-x+1)(3x^2) + (2x-1)(x^3+3)$$

3. $f(x) = \frac{x-1}{x^2-6x+2}$ $\qquad f'(x) = \frac{(x^2-6x+2)(1) - (x-1)(2x-6)}{(x^2-6x+2)^2}$

4. $y = \sqrt{x^2-6x} = (x^2-6x)^{1/2}$ $\qquad \frac{dy}{dx} = \frac{1}{2}(x^2-6x)^{-1/2}(2x-6)$

5. $y = \sqrt[3]{(x+\frac{1}{x})^2} = (x+x^{-1})^{2/3}$

$$\frac{dy}{dx} = \frac{2}{3}(x+x^{-1})^{-1/3}\left(1-\frac{1}{x^2}\right)$$

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(15) IV. On the following page carefully sketch the graph of

$$y = \frac{(x-1)^3}{(x+2)(x-5)^2},$$

and give the equation(s) of the asymptotes.

(20) V. In each of the following parts, find the equation of the straight line tangent to the curve at the given point.

1. $y = x^3 - 6x + 2$ at $(-1, 7)$
2. $x + 2xy - y^2 = 5$ at $(2, 3)$

1. $\frac{dy}{dx} = 3x^2 - 6$

$x = -1$

$m = 3 - 6 = -3$

$\frac{y-7}{x+1} = -3$

$y - 7 = -3(x+1)$

$y = -3x - 3 + 7$

$= -3x + 4$

$y = -3x + 4$

or $3x + y = 4$

2. $1 + 2x\frac{dy}{dx} + 2y - 2y\frac{dy}{dx} = 0$

$x = 2,\ y = 3$

$\frac{dy}{dx}$

$1 + 4\frac{dy}{dx} + 6 - 6\frac{dy}{dx} = 0$

$-2\frac{dy}{dx} = -7$

$\frac{dy}{dx} = \frac{7}{2}$

$\frac{y-3}{x-2} = \frac{7}{2}$

$y - 3 = \frac{7}{2}(x-2)$

$= \frac{7}{2}x - 7$

$y = \frac{7}{2}x - 4$

or ~~7x~~

$2y - 7x = -8$

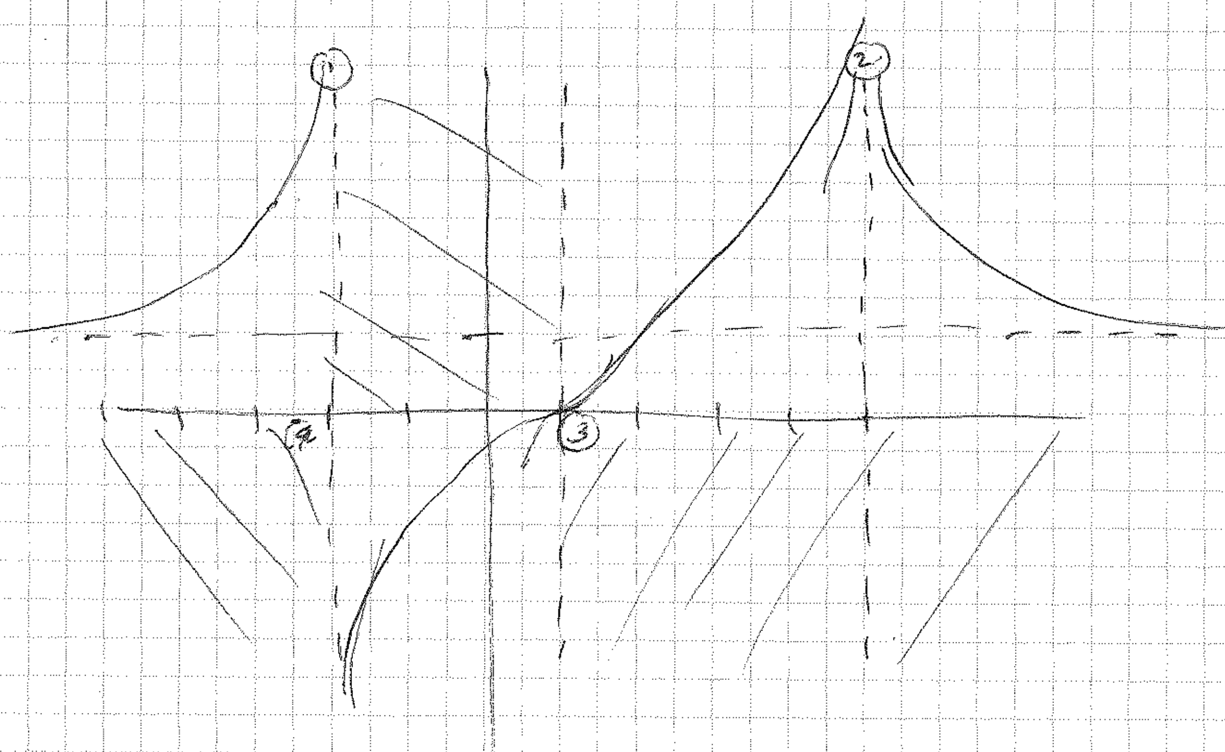
$(2x - 2y)\frac{dy}{dx} = -1 - 2y$

$X=1$ zero mult 3

(15) $X=-2,\ X=5$ poles

	1	2		2
	$(x+2)$	$(x-1)^3$	$(x-5)^2$	
$x<-2$	$-$	$-$	$+$	$+$
$-2<x<1$	$+$	$-$	$+$	$-$
$1<x<2$	$+$	$+$	$+$	$+$
$x>2$	$+$	$+$	$+$	$+$

$\frac{x^3}{x^3} \to 1$



Vert $x=2,\ x=5$

Hor $y=1$