Math 131
Test IV
Nov. 26, 1974

Name KEY

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(11) I. Let $z = x^3-6xy^2-3x^2y^3$. Find

1. $\frac{\partial z}{\partial x} = 3x^2-6y^2-6xy^3$

2. $\frac{\partial z}{\partial y} = -12xy-9x^2y^2$

3. $\frac{\partial^2 z}{\partial x \partial y} = -12y-18xy^2$

(15) II. 1. $\int x^3-6x+3\,dx = \frac{x^4}{4}-\frac{6x^2}{2}+3x+C = \frac{x^4}{4}-3x^2+3x+C$

2. $\int (6-2x)^7 dx = \frac{(6-2x)^8}{-2(8)}+C = -\frac{(6-2x)^8}{16}+C$

3. $\frac{dy}{dx} = x(x^2+1)^{5/2}$, solve for y.

$y = \frac{1}{2}\frac{(x^2+1)^{7/2}}{7/2}+C$

$= \frac{(x^2+1)^{7/2}}{7}+C$

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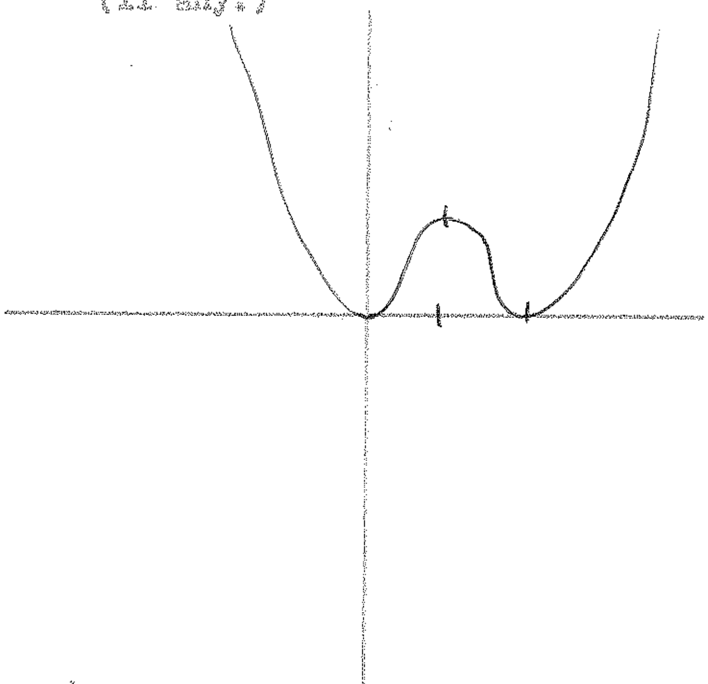
III. Sketch the graph of $y = x^4-4x^3+4x^2$. Include the following information: (Any methods may be used.) Hint: $y = x^2(x-2)^2$.

a. For what x is y an increasing function of x ?

b. Find the coordinates of all relative (local) maxima or minima (if any), and mark them on the graph with an M or m resp.

c. What are the coordinates of the absolute maxima or minima? (If any.)



a. incr

$0 < x < 1$

$\cancel{2}\ x > 2$

b. M $(1,1)$

m $(0,0)$

m $(2,0)$

$x=1 \quad y=1$

c. abs max none

abs min $(0,0)$ $(3,0)$

$$\frac{dy}{dx} = x^2\, 2(x-2) + 2x(x-2)^2$$

$$= 2x(x-2)\,(\overbrace{x + x-2}^{2x-2})$$

$$= 4x(x-2)(x-1)$$

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IV. Let $y = 3x^5+10x^4-20$.

a. For what values of x is the curve concave up?

b. For what values of x is the curve concave down?

c. Find the coordinates of all points of inflection (if any)?

$$\frac{dy}{dx} = 15x^4 + 40x^3$$

$$\frac{d^2y}{dx^2} = 60x^3 + 120x^2$$

$$= 60x^2(x+2)$$

a. $x > -2$

b. $x < -2$

c. I $(-2, 44)$

$3(-2)^5 + 10(-2)^4 - 20$

$-96 + 160 - 20$

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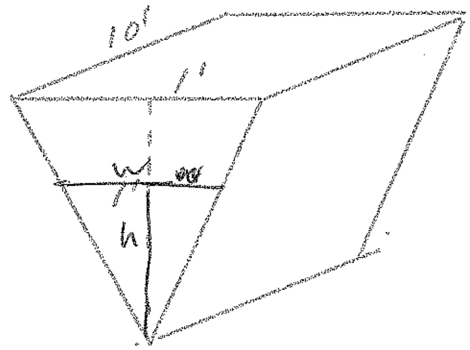
(13) V. A 10 foot long trough has triangular cross sections (isosceles) one foot wide by one foot deep. Water is being poured in at a rate of 2 cu. ft./min.

give units

a. At what rate is the water level rising when the trough is ~~"half full" (i.e. half the volume~~ $\frac{1}{2}$ 6" deep



$$\frac{dV}{dt} = 2$$

$$w = h$$

$$V = \frac{1}{2}h(w)\cdot 10$$

$$= 5h^2$$

$$\frac{5}{2} = 5h^2$$

$$\frac{1}{2} = h^2$$

$$h = \frac{1}{\sqrt{2}}$$

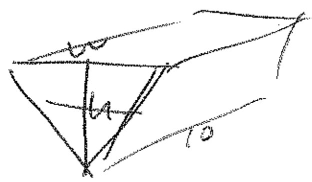
$$\frac{dV}{dt} = 20h\frac{dh}{dt}$$

$$2 = 20\left(\frac{1}{\sqrt{2}}\right)\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{2\sqrt{2}}{20} = \frac{\sqrt{2}}{10} \text{ ft/min}$$

(13) V.b. Keeping the volume fixed, what would the dimensions of the ends of the trough be to minimize the material used?

$5 = \cancel{5h}$ $\quad \frac{hw}{2}\cdot 10 = 5hw \qquad hw = 1 \quad w = \frac{1}{h}$



$$A = \frac{hw}{2} + \sqrt{(h)^2 + \left(\frac{w}{2}\right)^2}\cdot 10$$

$$= \frac{hw}{2} + \sqrt{h^2 + \frac{1}{4h^2}}\ 10$$

$$= \frac{1}{2} + 10\sqrt{h^2 + \frac{h^{-2}}{4}}$$

$$\frac{dA}{dh} = 10\ \frac{1}{2}\left(h^2 + \frac{h^{-2}}{4}\right)^{-1/2}\left(2h - \frac{2h^{-3}}{4}\right) = 0$$

$$2h - \frac{1}{2h^3} = 0$$

$\sqrt{2}\ w$

$\times\ \frac{1}{\sqrt{2}}$ high

$$2h = \frac{1}{2h^3} \qquad h^4 = \frac{1}{4}$$

$$h = \frac{1}{\sqrt{2}} \qquad w = \sqrt{2}$$

(13) VI. A car is moving at 30 ~~mi.~~ ft./~~hr.~~ sec. At a certain instant it is subjected to a constant acceleration for 3 sec., at the end of which it is going 42 ft./~~hr.~~ sec. How far did it travel in that 3 seconds?

$a = k$ $\qquad t=0 \quad v=30$

$\qquad t=3 \quad v=42$

$v = kt + c$ $\qquad t=0 \quad s=0$

$c = 30$

$v = kt + 30$

$42 = k3 + 30$

$3k = 12$

$k = 4$

$v = 4t + 30$

$s = 2t^2 + 30t + \underset{0}{\underset{\shortparallel}{c}}$

$s = 2t^2 + 30t$

$t = 3 \qquad s = 2(9) + 30(3)$

$\qquad = 18 + 90 = 108$ feet.

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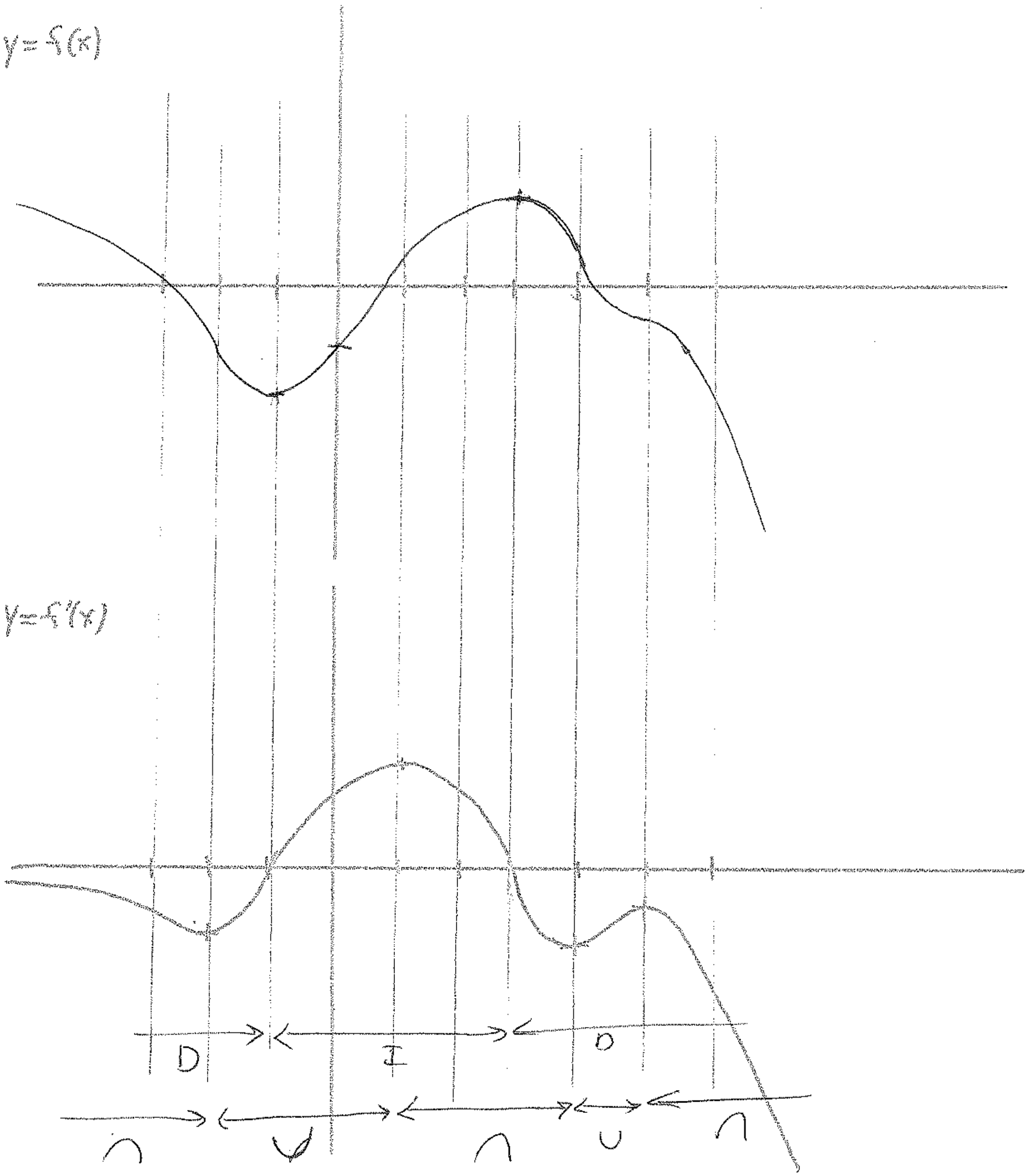
VII. Here is the graph of f'. Sketch a graph of f with $f(0)=-1$.



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