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F-9 min

Math 131
Test I
October 10, 1975 Name KEY

I. Short answer, 5 points each:

1. The slope of the line through the points (2,1) and (5,-3) is

$$\frac{7-(-3)}{5-2} = \frac{1+3}{5-2} = \frac{4}{3} \qquad \frac{-3-1}{5-2} = \frac{-4}{3}$$

2. The distance between the points (1,-2) and (3,7) is

$$\sqrt{(3-1)^2+(7-(-2))^2} = \sqrt{2^2+9^2} = \sqrt{85}$$

3. If a tank contains 100,000 gallons when the drain is opened, and 10 minutes later it is half empty; what is the average rate of change? (include the units)

$$\frac{50{,}000 - 100{,}000}{10} = -5000 \text{ gal/min}$$

(1)

4. The equation of the vertical line through (-3,-7) is

$$x = -3$$

5. The equation of the circle with center (7,-2) and radius 4 is

$$(x-7)^2+(y+2)^2 = 16$$

6. What is the slope of the line $x+2y+3 = 0$?

$$-\frac{1}{2}$$

7. At what point on the curve $y = x^2+2x+3$ is the tangent line horizontal?

$$y' = 2x+2 = 0$$
$$x = -1 \qquad y = (-1)^2+2(-1)+3 = 2$$

(-1)

(-1, 2)

8. The equation of the straight line through (3,4) which is perpendicular to the line $y = 4x-5$ is

$$m = -\frac{1}{4}$$

$$\frac{y-4}{x-3} = -\frac{1}{4}$$

$$y-4 = -\frac{1}{4}x+\frac{3}{4}$$

$$y = -\frac{1}{4}x+4\frac{3}{4}$$

$$x+4y = 19$$

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II. Using only the definition, find the derivative of each of the following

1. $f(x) = x^2 - 6x + 7$

2. $f(x) = \sqrt{3x+2}$

1. $f(x+\Delta x) = (x+\Delta x)^2 - 6(x+\Delta x) + 7$

$$\frac{\Delta y}{\Delta x} = \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 6x - 6\Delta x + 7 - (x^2 - 6x + 7)}{\Delta x}$$

$$= \frac{2x\Delta x + (\Delta x)^2 - 6\Delta x}{\Delta x} = 2x + \Delta x - 6$$

$$f'(x) = \lim_{\Delta x \to 0} (2x + \Delta x - 6) = 2x - 6$$

2. $$\frac{\Delta y}{\Delta x} = \frac{\sqrt{3(x+\Delta x)+2} - \sqrt{3x+2}}{\Delta x} \cdot \frac{\sqrt{3x+3\Delta x+2} + \sqrt{3x+2}}{\sqrt{\quad} + \sqrt{\quad}}$$

$$= \frac{3x + 3\Delta x + 2 - (3x+2)}{\Delta x\left(\sqrt{3x+3\Delta x+2} + \sqrt{3x+2}\right)} = \frac{3\Delta x}{\Delta x(\sqrt{\quad} + \sqrt{\quad})}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{3}{\sqrt{3x+3\Delta x+2} + \sqrt{3x+2}} = \frac{3}{2\sqrt{3x+2}} = \frac{3}{2}(3x+2)^{-1/2}$$

... 10 points.

Find the equation of the line tangent to the curve $y = 3x^2+x+2$ where it crosses the line $x = 2$.

$y' = 6x+1 \quad x=2$

$y = 3\cdot4+2+2 = 16$ $\qquad m = 13$

$\frac{y-16}{x-2} = 13 \qquad y-16 = 13x-26$

$y = 13x - 10$

IV. Carefully sketch the graph of each of the following: (30 points)

1. $y = 3x + 2$ 6
2. $y = -(x-2)(x+5)$ 6
3. $y = (x-1)(x+5)^2$ 8
4. $y = x^2(x+1)^4(x-5)^3(x-2)$ 8

Use this to show work. Use next page for graphs. Number them!

2. $x=2$ ① $\quad x=-5$ ①

	$x-2$	$x+5$	-1	y
$x<-5$	−	−	−	−
$-5<x<2$	−	+	−	+
$x>2$	+	+	−	−

3. mult

$x=1 \quad 1$

$x=-3 \quad 2$

	$x-1$	$(x+3)^2$	y
$x<-3$	−	+	−
$-3<x<1$	−	+	−
$x>1$	+	+	+

4. $x=-1$ ④

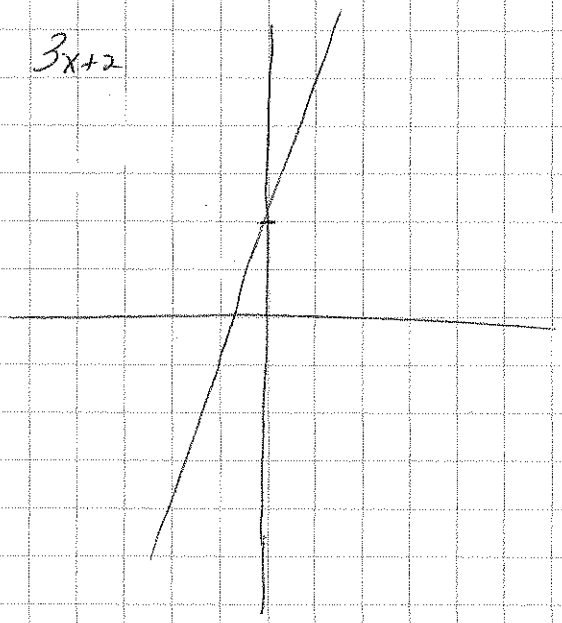
$x=0$ ②

$x=5$ ③

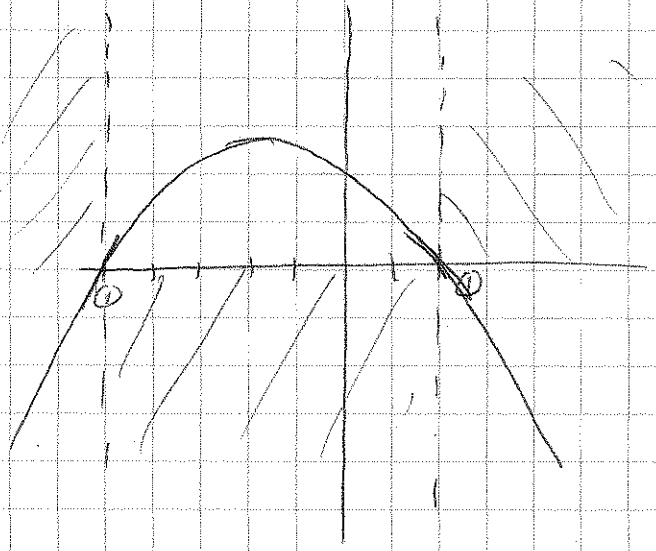
$x=2$ ①

	x^2	$(x+1)^4$	$(x-5)^3$	$x-2$	y
$x<-1$	+	+	−	−	+
$-1<x<0$	+	+	−	−	−
$0<x<2$	+	+	−	−	+
$2<x<5$	+	+	−	+	−
$x>5$	+	+	+	+	+

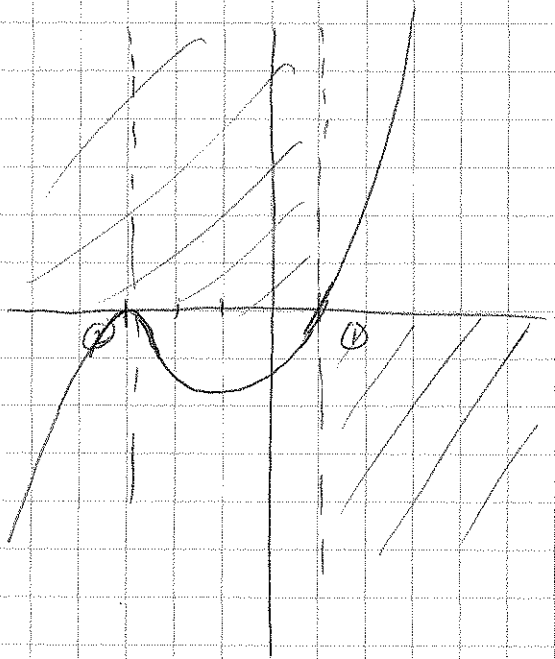
1. $3x+2$



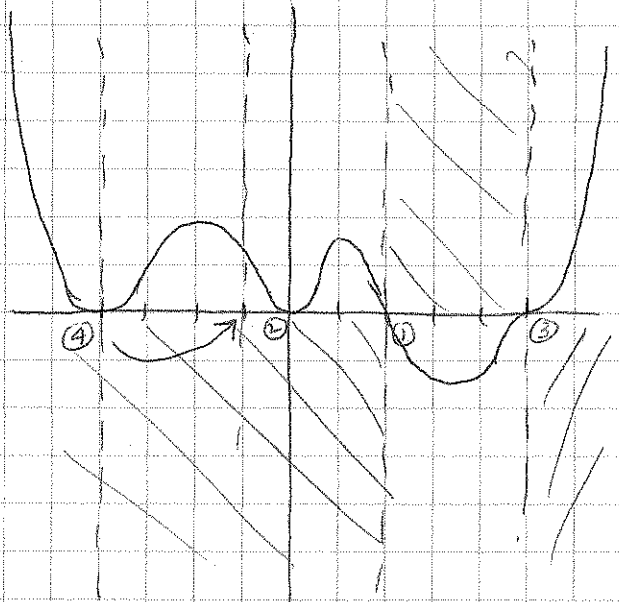
2. $-(x-2)(x+5)$



3. $(x-1)(x+3)^2$



4. $x^2(x+4)^4(x-5)^3(x-2)$



$x<-4$			
$-4<x$	$(x-5)^3$	$x-2$	
$x<2$	$-$	$-$	$+$
$2<x<5$	$-$	$+$	$-$
$x>5$	$+$	$+$	$+$