Math 131
Test II
October 31, 1975

Name KEY

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(35) I. Short Answer:

1. If $y = 2x^2 - 2x$, then $\frac{dy}{dx} =$ $4x - 2$

2. $\lim\limits_{x \to 2} \frac{x^2-1}{x+3} =$ $\frac{2^2-1}{2+3} = \frac{3}{5}$

3. $\frac{d}{dx}(x + \sqrt{x}) =$ $1 + \frac{1}{2}x^{-1/2} = 1 + \frac{1}{2\sqrt{x}}$

4. $\lim\limits_{x \to \infty} \frac{3x^3+x}{x^3-6x+1} =$ $\lim\limits_{x \to \infty} \frac{3 + 1/x^2}{1 - 6/x^2 + 1/x^3} = 3$

5. $\lim\limits_{x \to 1-} \frac{\overset{+}{x+2}}{\underset{-}{x-1}} =$ $-\infty$

6. $\frac{d}{dx}(2x^3 - 7x + \frac{2}{x}) =$ $6x^2 - 7 - 2x^{-2} = 6x^2 - 7 - \frac{2}{x^2}$

7. What are the vertical asymptotes (if any) of the curve $y = \frac{x}{x^2+3x+2}$?

$(x+2)(x+1)$

$x = -2, \; x = -1$

(10) II. Using limit theorems (one at each step), derive the following:

$$\lim_{x \to 3} \frac{(x-1)(x+2)}{2x^2+2} = \frac{\lim\limits_{x \to 3} (x-1)(x+2)}{\lim\limits_{x \to 3} (2x^2+2)} = \frac{\lim\limits_{x \to 3} (x-1) \lim\limits_{x \to 3} (x+2)}{\lim\limits_{x \to 3} (2x^2+2)}$$

$$= \frac{\left(\lim\limits_{x \to 3} x + \lim\limits_{x \to 3} (-1)\right)\left(\lim\limits_{x \to 3} x + \lim\limits_{x \to 2} 2\right)}{\lim\limits_{x \to 3} 2x^2 + \lim\limits_{x \to 3} 2}$$

$$= \frac{(3 + (-1))\,(3+2)}{2 \lim\limits_{x \to 3} x^2 + \cancel{\lim\limits_{x \to 3}} 2}$$

$$= \frac{(3 + (-1))(3+2)}{2 \cdot 9 + 2} = \frac{2 \cdot 5}{\underset{4}{\cancel{20}}} = \frac{1}{2}$$

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1. Find $\frac{dy}{dx}$, show work ! Dont : simplify algebra.

1. $y = (x^3-6x)(2x^4+2x^2-1)$

$$\frac{dy}{dx} = (x^3-6x)(8x^3+4x) + (2x^4+2x^2-1)(3x^2-6)$$

2. $y = (3x^2+5)^3$

$$\frac{dy}{dx} = 3(3x^2+5)^2(6x)$$

3. $y = \frac{x+3}{x^2-1}$ $\qquad \frac{dy}{dx} = \frac{(x^2-1)\cdot 1 - (x+3)(2x)}{(x^2-1)^2}$

4. $y = \sqrt{3x-1}$ $\qquad \frac{dy}{dx} = \frac{1}{2}(3x-1)^{-1/2}(3)$

$= (3x-1)^{1/2}$

5. $y = \sqrt{(x-\frac{1}{x})^3}$ $\qquad \frac{dy}{dx} = \frac{3}{2}(x-x^{-1})^{1/2}(1+x^{-2})$

$= (x-x^{-1})^{3/2}$

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6. $x^3 + 2xy^2 = 2$

$$3x^2 + 2x2y\frac{dy}{dx} + 2y^2 = 0$$

$$\frac{dy}{dx} = \frac{-3x^2-2y^2}{4xy}$$

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(10) IV. Graph $y = \dfrac{2x(x+3)^2}{(x-1)(x-4)^2}$ on the next page.

(5) V. 1. Approximate $\sqrt{8.9}$.

$$y = \sqrt{x} \qquad x = 9$$
$$\Delta x = -.1$$

$$\Delta y \approx \frac{1}{2\sqrt{x}} \Delta x$$
$$\approx \frac{1}{2\sqrt{9}}(-.1) = -\frac{.1}{6}$$

$$y + \Delta y \approx 3 - \frac{.1}{6}$$
$$\approx 3 - .0166\overline{6}$$
$$= 2.983\overline{3}$$

$2\frac{59}{60}$

2. Find the equation of the straight line tangent to the curve $x^2y + y^2 = 6$ at $(1,2)$.

$$x^2 \frac{dy}{dx} + 2xy + 2y\frac{dy}{dx} = 0$$

$(1,2)$

$$1\frac{dy}{dx} + 4 + 4\frac{dy}{dx} = 0$$

$$5\frac{dy}{dx} = -4$$

$$\frac{dy}{dx} = -\frac{4}{5}$$

$$\frac{y-2}{x-1} = -4/5$$

$$y = -4/5\,x + 4/5 + 2$$
$$= -4/5\,x + \frac{14}{5}$$

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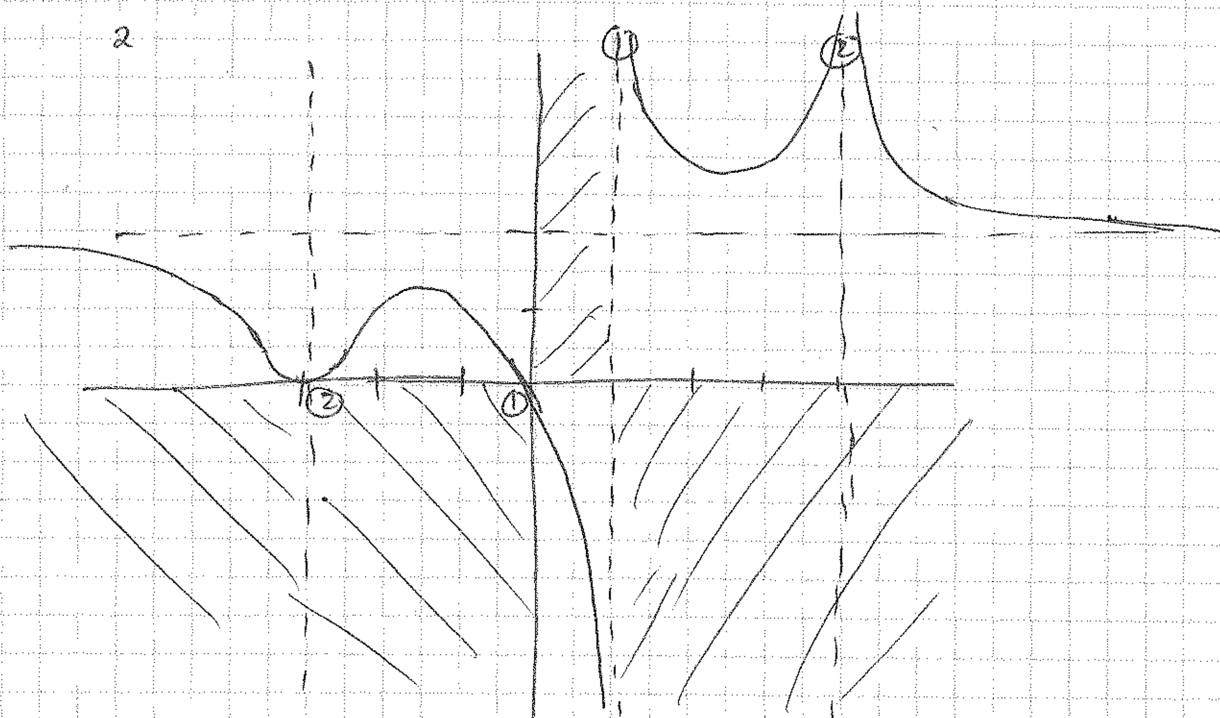
$$y = \frac{2x\,(x+3)^2}{(x-1)\,(x-4)^2}$$

Zeros	mult
$x=0$	1
$x=-3$	2
Poles	**order**
$x=1$	1
$x=4$	2

	$2x$	$(x+3)^2$	$x-1$	$(x-4)^2$	y
$x<-3$	$-$	$+$	$-$	$+$	$+$
$-3<x<0$	$-$	$+$	$-$	$+$	$+$
$0<x<1$	$+$	$+$	$-$	$+$	$-$
$1<x<4$	$+$	$+$	$+$	$+$	$+$
$x>4$	$+$	$+$	$+$	$+$	$+$



$$\frac{2x\,(x+3)^2}{(x-1)(x-4)^2} \longrightarrow 2$$

$$\frac{2x\,(x+3)^2}{(x-1)(x-4)^2} = 2$$

$$2x(x^2+6x+9) = 2(x-1)(x^2-8x+16)$$

$$\cancel{x^3} - 8x^2 + 16x - x^2 + 8x - 16$$

$$\cancel{2x^3} + 12x^2 + 9x = \cancel{2x^3} - 18x^2 - 16x - 32$$

$$30x^2 + 25x + 32 = 0$$

$$x = \frac{-25 \pm \sqrt{25^2 - 4 \cdot 30 \cdot 32}}{60} \quad \text{no pts.}$$