Math 131

Test III

Nov. 25, 1975 Name KEY

802
12

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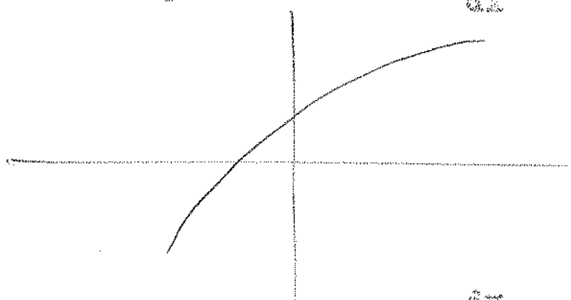
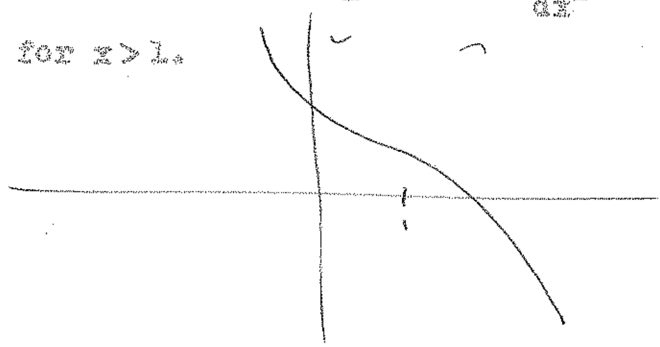
med = 57
hi 86
lo 12

5 ea. I. Short Answer.

1. Draw a graph for which $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$.



2. Draw a graph for which $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} > 0$ for $x < 1$ and $\frac{d^2y}{dx^2} < 0$ for $x > 1$.



3. If $f(x) = \frac{x^2(x-1)}{(x-2)(x+3)}$, for which x is f discontinuous?

$x = 2, x = -3$

4. $\int x^2 + \sqrt{x}\, dx = \frac{x^3}{3} + \frac{2}{3}x^{3/2} + C$

5. $\int x\sqrt{x^2+4}\, dx = \frac{(x^2+4)^{3/2}}{3/2 \cdot 2} + C = \frac{(x^2+4)^{3/2}}{3} + C$

6. Solve: $\frac{dy}{dx} = 2xy^2$

$y^{-2}dy = 2x\,dx$

$\frac{y^{-1}}{-1} = x^2 + C$

$-\frac{1}{y} = x^2 + C$

30

10 I. 7. A. If a function is continuous, it has a derivative.

B. If a function has a derivative, it is continuous there.

( ) Both true ( ) A true, B false (✓) B true, A false ( ) Both false.

8. A. If the derivative of a function is zero or does not exist, then there is a maximum or minimum at that point.

B. If there is a maximum or minimum at a point, then the derivative is zero of does not exist.

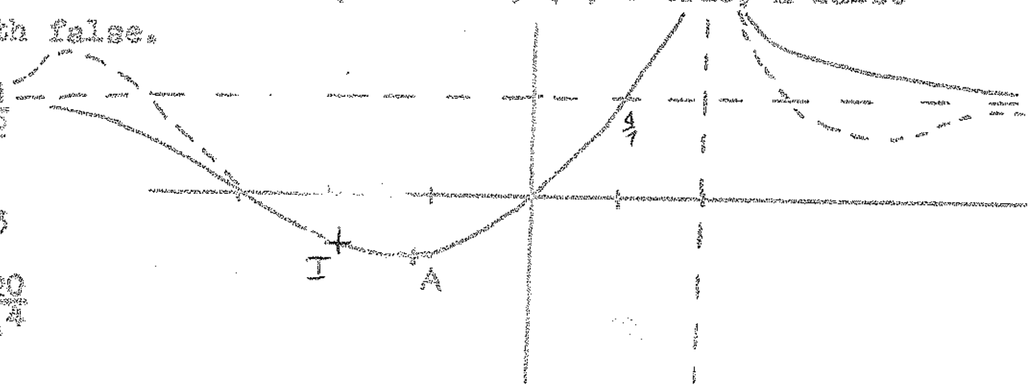
( ) Both true ( ) A true, B false, (✓) B true, A false

( ) Both false.

25 II. $y = \frac{x(x+3)}{(x-2)^2}$

$\frac{dy}{dx} = \frac{-7x-6}{(x-2)^3}$

$\frac{d^2y}{dx^2} = \frac{14x+20}{(x-2)^4}$



1. Find the coordinates of the minimum point A.
2. For what x is graph increasing? decreasing?
3. Find (and mark on graph) any points of inflection.
4. Give reasons why graph is NOT the dotted line.

1. $\frac{dy}{dx} = 0 \quad -7x+6=0 \quad x = -\frac{6}{7} \qquad \left(-\frac{6}{7}, -\frac{9}{40}\right)$

$y = \frac{-6/7\left(-\frac{6}{7}+3\right)}{\left(-\frac{6}{7}-2\right)^2} = \frac{-6(-6+21)}{(-6-14)^2}$

$= \frac{-6(15)}{(20)^2} = -\frac{9}{40}$

20 . 42

2. I $-6/40 < x < 2$

D $x < -6/40, \ x > 2$

3. $14x+20=0$

$x = -\frac{10}{7}$

$y = \frac{-\frac{10}{7}\left(-\frac{10}{7}+3\right)}{\left(-\frac{10}{7}-2\right)^2}$

$= \frac{-10(-10+21)}{(-10-14)^2}$

$= \frac{-10(11)}{(-24)^2} = \frac{-10 \cdot 11}{12 \cdot 2 \cdot 24} = \frac{-55}{288} \approx \frac{1}{5}$

4. no more critical pts or does not cross y=1 more than once

$1 = \frac{x(x+3)}{(x-2)^2}$

$(x-2)^2 = x^2+3x$

$x^2-4x+4 = x^2+3x$

$4 = 7x$

$x = \frac{4}{7}$

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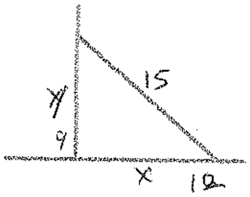
III.

5 1. A fifteen foot ladder is resting against a wall. The bottom is being pulled out at the rate of 3 ft./sec. How fast is the top falling when the top is 9 ft. from the ground?



$\frac{dx}{dt} = 3$

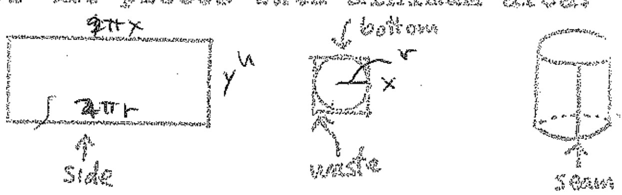
$x^2 + y^2 = 15^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{x\frac{dx}{dt}}{y} = -\frac{12}{9}\cdot 3 = -4$

4 ft/sec

20 2. A cylindrical can with no top is to be made from these pieces. If the volume is to be 16 cu.in. , what are the dimensions of the pieces with minimum area?



$16 = \pi\left(\frac{x}{2}\right)^2 \cdot y = \frac{\pi x^2 y}{4}$ $\qquad y = \frac{4}{\pi}x^{-2}$

$A = 2\pi x y + x^2$

$A = 2\pi x \frac{4}{\pi}x^{-2} + x^2$ $\qquad 2\sqrt[3]{4}\pi \times \frac{\sqrt[3]{4}}{\pi}$ $\qquad \sqrt[3]{4}\times\sqrt[3]{4}$

$= 8x^{-1} + x^2$ $\qquad$ or $\quad A = 2\pi r h + 4r^2$

$\frac{dA}{dx} = -8x^{-2} + 2x = 0$ $\qquad 16 = \pi r^2 h$

$h = \frac{16}{\pi}r^{-2}$

$4 = x^3$

$x = \sqrt[3]{4}$ $\qquad A = 2\pi r\frac{16}{\pi}r^{-2} + 4r^2$

$= 32r^{-1} + 4r^2$

$y = \frac{4}{\pi}4^{-2/3}$ $\qquad \frac{dA}{dr} = -32r^{-2} + 8r = 0$

$= \frac{\sqrt[3]{4}}{\pi} =$ $\qquad 4 = r^3$

$r = \sqrt[3]{4}$

$2\sqrt[3]{4}$ sq $\qquad h = \frac{16}{\pi}4^{-2/3} = \frac{4^{4/3}}{\pi}$

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Second try (Part of Final Exam)

Name KEY

Other part was Calc I Placement test

5 ea I. Short Answer.

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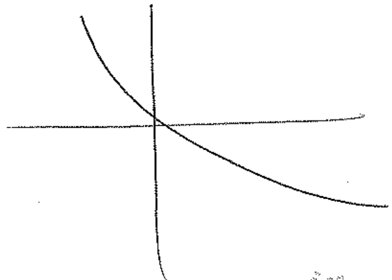
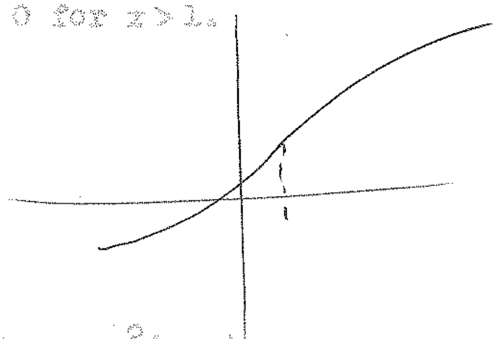


2. Draw a graph for which $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$ for $x < 1$ and $\frac{d^2y}{dx^2} < 0$ for $x > 1$.



3. If $f(x) = \frac{x^2(x+1)}{(x+2)(x+3)}$, for which x is f discontinuous?

$x = -2,\ x = -3$

4. $\int x^2 + \sqrt{x}\ dx =$ $\quad x^{1/2}$ $\quad \frac{x^3}{3} + \frac{2}{3}x^{3/2} + c$

5. $\int 3x\sqrt{x^2+4}\ dx =$ $\quad \frac{3}{2}\frac{(x^2+4)^{3/2}}{3/2} + c = (x^2+4)^{3/2} + c$

6. Solve: $\frac{dy}{dx} = 2xy^2$

$$y^{-2}dy = 2x\,dx$$

$$\frac{y^{-1}}{-1} = x^2 + c$$

$$-\frac{1}{y} = x^2 + c$$

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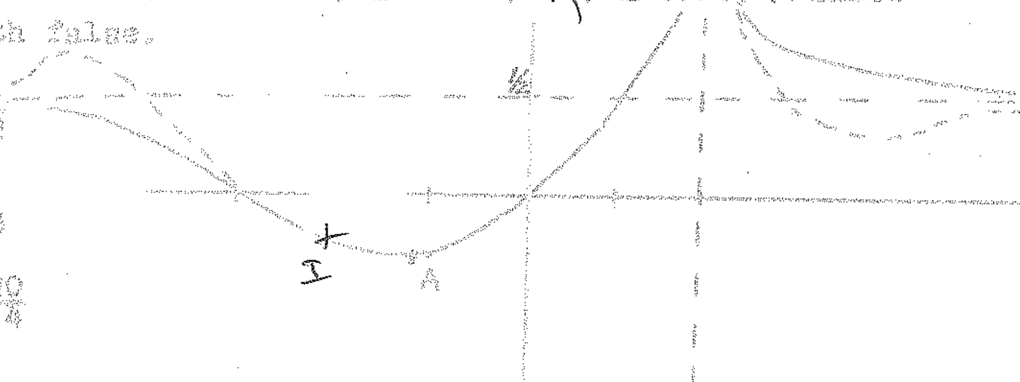

1. Find the coordinates of the minimum point A.
2. For what x is graph increasing? concave up?
3. Find coordinates (and mark on graph) ~~any~~ all points of inflection.
4. Give reasons why graph is NOT the dotted line.

1. ~~x~~ $-7x-6=0$

$x = -\frac{6}{7}$

$y = \dfrac{-\frac{6}{7}\left(-\frac{6}{7}+3\right)}{2\left(-\frac{6}{7}-2\right)^2} = \dfrac{-6\left(-6+21\right)}{2\left(-6-14\right)^2} = \dfrac{-6(15)}{2\cdot 400} = -\dfrac{9}{80}$

$\left(-\frac{6}{7}, -\frac{9}{80}\right)$

2. $-\frac{6}{7} < x < 2$ inc

~~x~~ $-\frac{10}{7} < x < 2$

$x > 2$

4. $y \neq \frac{1}{2}$

or no more CD

or no more PI

3. $x = -\frac{10}{7}$

$y = \dfrac{-\frac{10}{7}\left(-\frac{10}{7}+3\right)}{2\left(-\frac{10}{7}-2\right)^2} = \dfrac{-10(-10+21)}{2(-10-14)^2}$

$= \dfrac{-5(11)}{(-24)^2} = -\dfrac{55}{576}$

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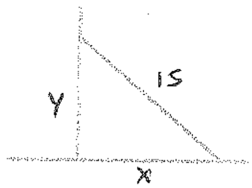
III.

15 1. A fifteen foot ladder is resting against a wall. The bottom is being pulled out at the rate of 4 ft./sec. How fast is the top falling when the top is 9 ft. from the ground?



$\frac{dx}{dt} = 4 \qquad x^2 + y^2 = 15^2$

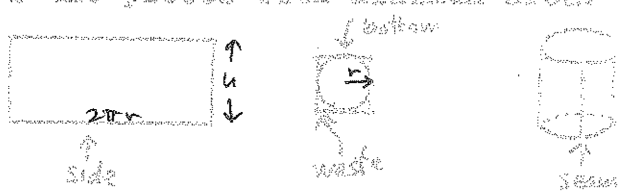
Find $\frac{dy}{dt}$ when $y = 9. \qquad x = 12$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$, with $\frac{dx}{dt} = 4$

$\frac{dy}{dt} = -\frac{4x}{y} = -\frac{4(12)}{9} = -\frac{48}{9} = -\frac{16}{3} = -5\frac{1}{3}$

$5\frac{1}{3}$ ft/sec.

20 2. A cylindrical can with no top is to be made from these pieces. If the volume is to be 32 cu.in. , what are the dimensions of the pieces with minimum area?



$A = 2\pi r h + (2r)^2 \qquad 32 = \pi r^2 h \qquad h = \frac{32}{\pi r^2}$

over r

$A = 2\pi r \frac{32}{\pi} r^{-2} + 4r^2$

$= 64 r^{-1} + 4r^2$

$\frac{dA}{dr} = -64 r^{-2} + 8r = 0$

$8r^3 = 64$

$r^3 = 8$

$r = 2$

$h = \frac{32}{\pi 4} = \frac{8}{\pi}$

4×4 sq $\qquad \frac{8}{\pi} \times 4\pi$ rect.

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