Math 131
Final Exam
December 5, 1978

Name KEY
100( ) 150( )

Started leaving after $1\frac{1}{2}$ hr

Show work!

(10) 1. Find the following limits:

a. $\lim_{x\to\infty} \frac{2x^2-3}{5x(x-5)} = \frac{2}{5}$

b. $\lim_{x\to 0^+} \frac{3}{5x(x-5)} = \frac{3}{+(-5)} = -\infty$

(30) 2. Find the derivative y', $\frac{dy}{dx}$ in each of the following:

a. $y = x^6 - 6\sqrt{x} - \frac{8}{x^5} + x\sqrt{x} = x^6 - 6x^{1/2} - 8x^{-5} + x^{3/2}$

$$\frac{dy}{dx} = 6x^5 - 3x^{-1/2} + 40x^{-6} + \frac{3}{2}x^{\frac{1}{2}}$$

b. $y = (x^3-6)^5$

$$y' = 5(x^3-6)^4(3x^2)$$

c. $y = \sin x - x \sin x$

$$y' = \cos x - (x\cos x + \sin x)$$
$$= \cos x - x\cos x - \sin x$$

d. $y = \cos(x^3-2)$

$$y' = -\sin(x^3-2)\, 3x^2$$

e. $y = \sin^2 x$ $\quad y' = 2\sin x \cos x$

f. $x^2y^2 + y = 3$

$$x^2 2yy' + 2xy^2 + y' = 0$$
$$2x^2yy' + y' = -2xy^2$$
$$y' = \frac{-2xy^2}{2x^2y+1}$$

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(10) 3. Find the derivative of $f(x) = 3x^2$ using the definition.

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{3(x+\Delta x)^2 - 3x^2}{\Delta x} = \frac{3[\cancel{x^2}+2x\Delta x + \Delta x^2 - \cancel{x^2}]}{\Delta x}$$

$$= \frac{6x\Delta x + 3\Delta x^2}{\Delta x} = 6x + 3\Delta x$$

$$\lim_{\Delta x \to 0}(6x + 3\Delta x) = 6x$$

(10) 4. Find the equation of the line tangent to the curve $y = \frac{3}{x-4}$ at (7, 1)

$y = 3(x-4)^{-1}$ $\quad y' = -3(x-4)^{-2}$

$x = 7$ $\quad y' = \frac{-3}{3^2} = -\frac{1}{3}$ $\qquad \frac{y-1}{x-7} = -\frac{1}{3}$

$y = -\frac{1}{3}x + \frac{7}{3} + 1$

$= -\frac{1}{3}x + \frac{10}{3}$

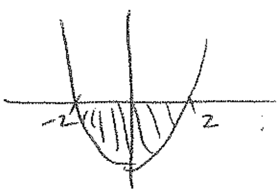
(15) 5. Evaluate the following integrals:

a. $\int x^2 - 3x\,dx = \frac{x^3}{3} - \frac{3x^2}{2} + C$

b. $\int \cos x\,dx = \sin x + C$

c. $\int_{-1}^{2} 2x^3\,dx = \frac{2x^4}{4}\Big|_{-1}^{2} = \frac{2\cdot 2^4}{4} - \frac{2(-1)^4}{4} = 8 - \frac{1}{2} = 7\frac{1}{2}$

(10) 6. Find the area bounded by the curve $y = x^2 - 4$ and the x-axis (between the points where they cross).



$$-\int_{-2}^{2} x^2 - 4\,dx = -\left(\frac{x^3}{3} - 4x\right)\Big|_{-2}^{2}$$

$$= -\left(\frac{8}{3} - 8\right) + \frac{-8}{3} + 8 = 16 - \frac{16}{3} = \frac{32}{3}$$

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7. A projectile is fired up into the air. Its position is given by $s = -16t^2 + 800t$ feet from the ground after t seconds.

a. What is the velocity after 5 seconds? Is it going up or down then?
b. When does it hit the ground?
c. What is the maximum height that it reaches?

$$\frac{ds}{dt} = -32t + 800$$

a. $t = 5$ $\quad V = -32(5) + 800 = 800 - 160 = 640$ ft/sec up

b. $S = 0$ $\quad -16t^2 + 800 = 0$ $\quad \frac{800}{16} = 50$

$-16t(t-50)$

$t = 0,\ t = 50$

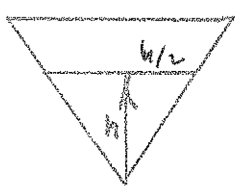
c. $-32t + 800 = 0$

$t = \frac{800}{32} = \boxed{25}$ $\quad -16(25)^2 + 800(25)$

$-16 \cdot 625 + 20{,}000$

$-10{,}000 +$ $\quad = 10{,}000$

(10) 8. Suppose that a conical tank is shaped so that it is 10 feet deep and 10 feet across the top. Water is being drained out at the rate of 2 cu. ft. per hr. At what rate is the water level falling when the water is 5 ft. deep?



$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h \qquad 0 \le h \le 10$$

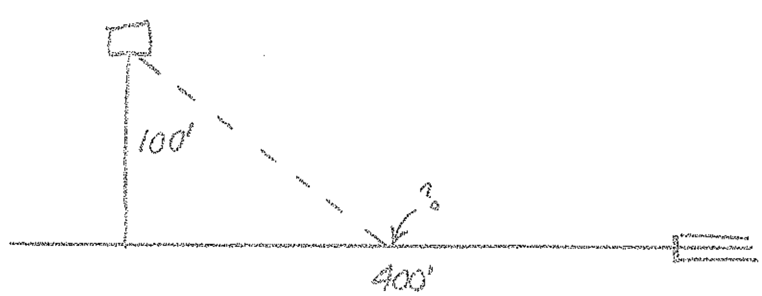
$$= \frac{1}{3}\pi\frac{h^3}{4} = \frac{\pi}{12}h^3 \qquad \frac{dV}{dt} = -2$$

$$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$$

$$-2 = \frac{\pi}{4}(5)^2\frac{dh}{dt} \implies \frac{dh}{dt} = \frac{-2 \cdot 4}{25 \cdot \pi} = -\frac{8}{25\pi} \text{ ft/sec}$$

≈ -10

(10) 9. (Problem that was taken home to do.)
The building in the picture is 100 feet from the road at a point 400 feet from the end of the water main. Pipe layed along the road costs 50 dollars a foot while pipe across open ground costs 70 dollars a foot. At what point on the road should we begin to lay pipe across the open area so as to minimize the total cost?



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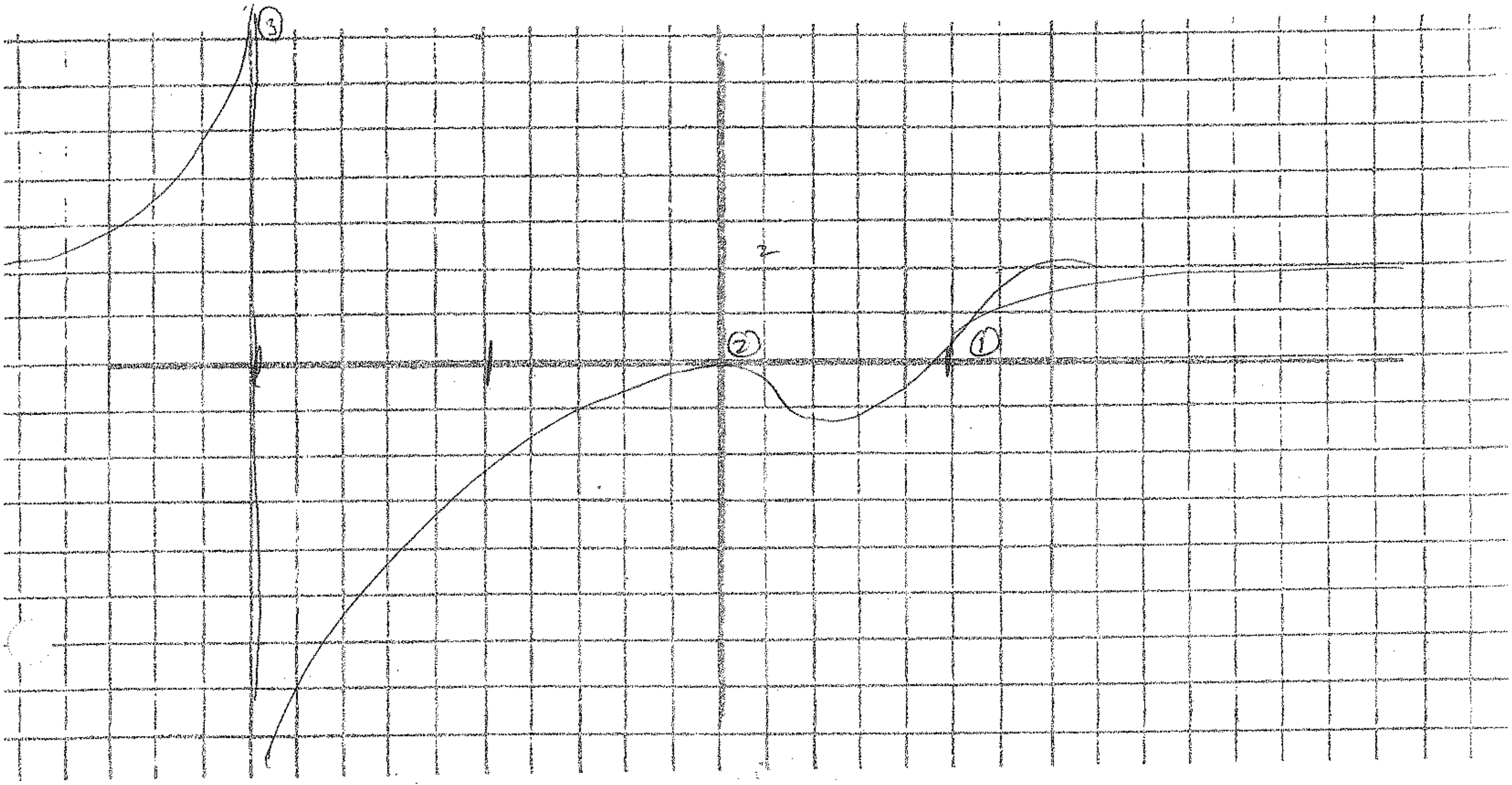
(15) 10. Sketch the graph of $y = \dfrac{2x^2(x-1)}{(x+2)^3}$. You need not use calculus. Draw and givethe equations of all asymptotes. $y=2$ $x=-2$

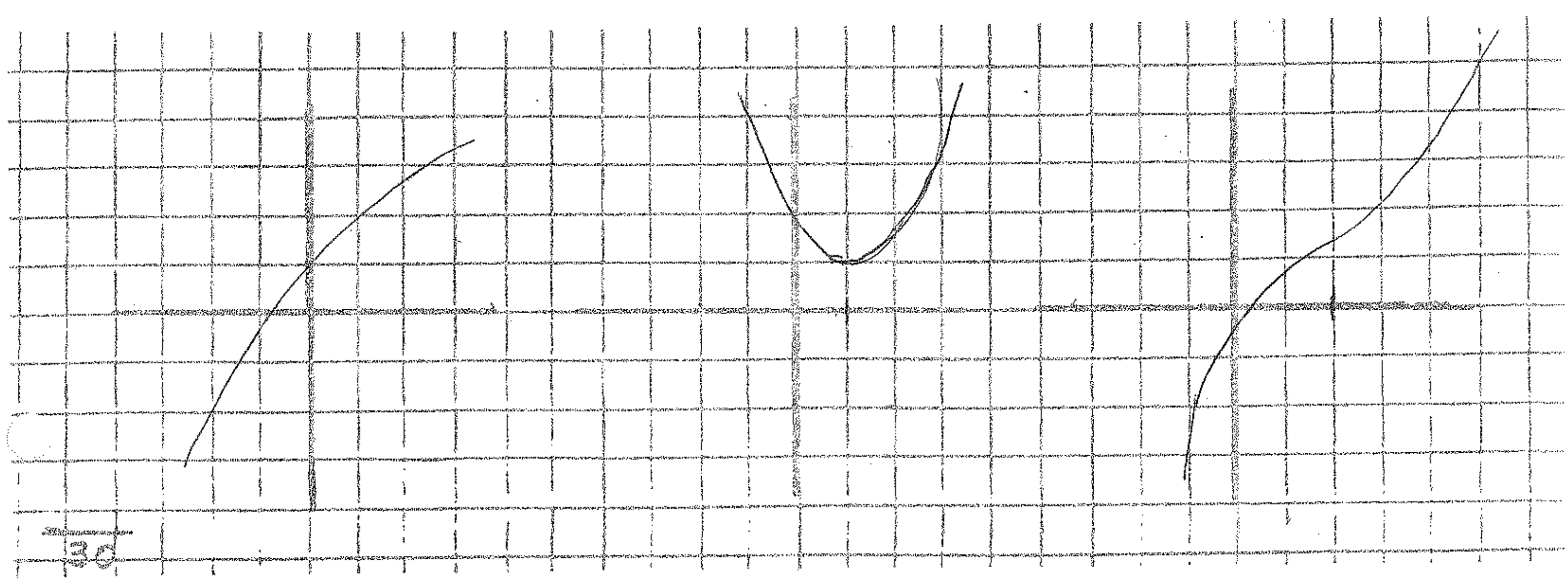
Extra Credit: Find the coordinates of the relative maxima and minima.
More Extra Credit: Find the points of Infelection.



(15) 11. a. Draw a graph for which $f'(x) > 0$ and $f''(x) < 0$ for all x.

b. Draw a graph for which $f'(x) > 0$ for $x > 1$, $f'(x) < 0$ for $x < 1$, and $f'(1) = 0$.

c. Draw a graph with $f''(x) < 0$ for $x < 2$, $f''(x) > 0$ for $x > 2$.



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Der

$$\frac{(x+2)^3(3x^2-2x)-(x^3-x^2)3(x+2)^2}{(x+2)^6}$$

$$\frac{(x+2)^2\left[(x+2)(3x^2-2x)-3(x^3-x^2)\right]}{(x+2)^6}$$

$$3x^3+6x^2-2x^2-4x-3x^3+3x^2$$

$$\frac{7x^2-4x}{(x+2)^4}=\frac{x(7x-4)}{(x+2)^4}$$

$x=\frac{4}{7}$

0

2nd der

$$\frac{(x+2)^3(3x^2-1)-(x^3-x)3(x+2)^2}{(x+2)^3}$$

$$3x^3+6x^2-x-2-3x^3+3x$$

$$\frac{(x+2)^2\left[(x+2)(3x^2-1)-3(x^3-x)\right]}{(x+2)^3}$$

$$\frac{6x^2+2x-2}{(x+2)}$$

$$\frac{\frac{49}{16}\left(-\frac{3}{4}\right)}{\left(\frac{15}{4}\right)^3}=\frac{-49}{15^2\cdot 5}=\frac{-49}{1125}$$

$$\frac{-2\frac{2}{5}}{1\frac{3}{25}}$$

0

$6x$

$$-2\pm\sqrt{4-4(6)(-2)}$$

$$-2\pm\sqrt{4+48}$$

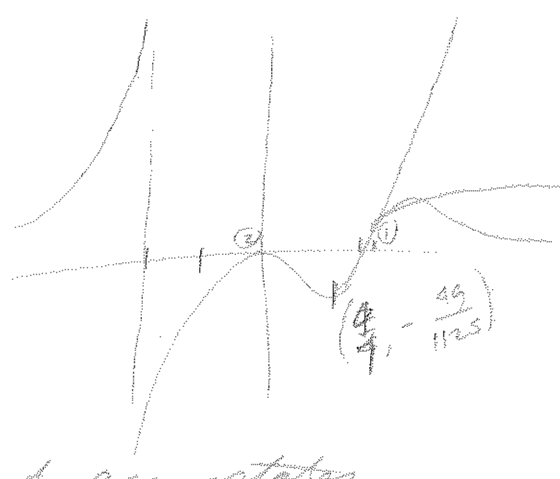
$$-2\pm\sqrt{52}$$

$$\frac{-2\pm2\sqrt{13}}{12}$$

(10-5) Sketch the graph of $f(x)=$

You need not use calculus

$$\frac{2x^2(x-1)}{(x+2)^3}\qquad\frac{x^3-x^2}{(x+2)^3}$$



Use: ~~zeros~~ eq of hor + vert asymptotes.

10?

$$\frac{(x+2)^4(14x-4)-(7x^2-4x)(x+2)^3\,4}{(x+2)^8}$$

$$\frac{(x+2)^3\left[(x+2)(14x-4)-(7x^2-4x)\,4\right]}{(x+2)^{\cancel{8}\,5}}$$

$14x^2+28x-4x-8-28x^2+16x$

$-14x^2+40x-8$

$-7x^2+20x-4$

$$\frac{-20\pm\sqrt{400-\overset{28}{\underset{112}{\cdot 4}}}}{-14}$$

$$\frac{-20\pm\sqrt{288}}{-14}$$ $=12\sqrt{2}$ $\quad \frac{10}{7}\pm\frac{6\sqrt{2}}{7}$

$(2.64, .2288)$ ≈ 1.5

$(.214, -.0066)$