Math 131
Quiz II
Nov. 10, 1978

Name KEY

Show work!

(15) I. Find the derivative of each of the following:

1. $y = (3x^2-6)^7$ 2. $y = 3\sin^2 x + \cos x - 2x$ 3. $y = \sin(2x) + \sqrt{\cos x + 1}$

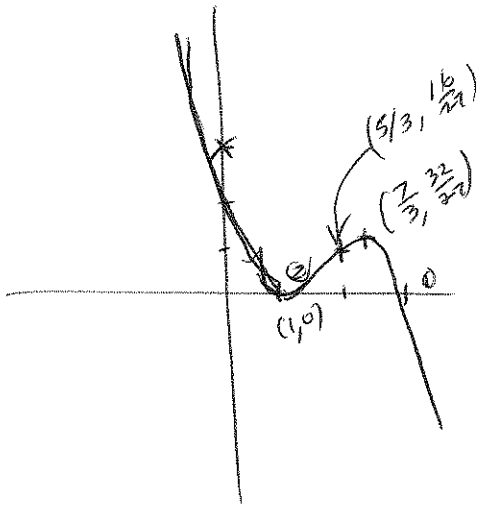
$\frac{dy}{dx} = 7(3x^2-6)^6 6x$
$= 42x(3x^2-6)^6$

$\frac{dy}{dx} = 6\sin x\cos x - \sin x - 2$

$\frac{dy}{dx} = \cos 2x \cdot 2 + \frac{1}{2}(\cos x+1)^{-1/2}(-\sin x)$
$= 2\cos 2x - \frac{\sin x}{2\sqrt{\cos x+1}}$

(25) II. Let $f(x) = (x-1)^2(3-x)$.

1. Graph the curve.
2. Find the coordinates of all max and min points.
3. Find the coordinates of all points of inflection.
4. For what valuese is f increasing? decreassing ?
5. For what values concave up? down?
6. What is the absolute maximum for $0 \le x \le 3$? min ?



$f' = 2(x-1)(3-x) + (x-1)^2(-1)$
$= (x-1)(2(3-x) - (x-1))$
$= (x-1)(6-2x-x+1)$
$= (x-1)(7-3x)$
$x = 1, \quad x = \frac{7}{3}$

$f(\frac{7}{3}) = (\frac{4}{3})^2(\frac{2}{3}) = \frac{32}{27}$ MAX $(\frac{7}{3}, \frac{32}{27})$

I $1 \le x \le \frac{7}{3}$
D $x > \frac{7}{3}$
$x < 1$

$f'' = (x-1)(-3) + (7-3x)$
$= -3x + 3 + 7 - 3x$
$= 10 - 6x$ $8-4x$

$f(\frac{10}{3})$ $(2,1)$
$f(2) = 2$

CC↓ $x > \frac{5}{3}$
CC↑ $x < \frac{5}{3}$

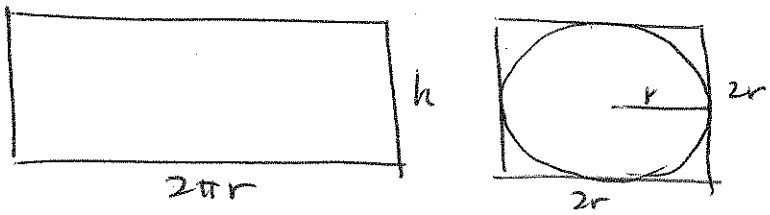
$x = \frac{10}{6}$ $x = 2$
$-3x + 3 + 7 - 3x$
$10 - 6x$ $x = \frac{10}{6} = \frac{5}{3}$

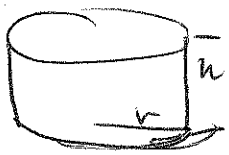
$f(\frac{5}{3}) = (\frac{2}{3})^2(\frac{2}{3}) = \frac{16}{27}$

6. (2)
0

III. A can is to be made from a rectangular piece and a square piece of metal. The can has no top, and the bottom is to be cut out of the square piece. If the volume is to be 32 cm^3 (cc), what size should the pieces of metal be so that the total amount of material (incl waste) is a minimum?

(11)





$V = \pi r^2 h = 32$

$h = \frac{32}{\pi r^2}$

$M = 2\pi r h + 4r^2$

$M = 2\pi r \frac{32}{\pi r^2} + 4r^2$

$= 64 r^{-1} + 4r^2$

$\frac{dM}{dr} = -64 r^{-2} + 8r$

$\frac{8(8 + r^3)}{r^2}$ $r = 2$

$h = \frac{8}{\pi}$

4 x 4 square

$\frac{8}{\pi}$ x 4π, rect.