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Math 131
Test I
Oct. 20, 1978

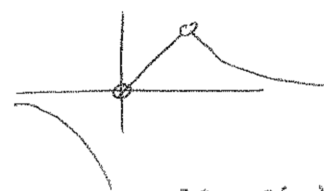
Name *KEY*

(10) 1. Let $f(x) = \frac{x^2-1}{2x(x-1)}$. Find:

a. $\lim_{x \to 1} f(x) = 1$ b. $\lim_{x \to \infty} f(x) = \frac{1}{2}$ c. $\lim_{x \to 0^+} f(x) = +\infty$

$\frac{x+1}{2x} \to \frac{2}{2}$ $\frac{x^2}{2x^2}$ $\frac{-1}{2+\ -1}$

(15) 2. Let $f(x) = \begin{cases} \frac{1}{x}, & x < 0,\ x > 1 \\ x, & 0 < x < 1 \\ k, & x = 1 \end{cases}$. Find:



a. $\lim_{x \to 0^-} f(x) = -\infty$ b. $\lim_{x \to 0^+} f(x) = 0$ c. $\lim_{x \to \infty} f(x) = 0$

d. What value of k will make the function continuous at x = 1 ?

$k = 1$

e. Are there any other points of discontinuity? If so which ones?

$x = 0$

(5) 3. If $f(x) = \frac{x - g(x)}{h(x)}$, $\lim_{x \to 2} g(x) = 1$, $\lim_{x \to 2} h(x) = 3$, then ~~lim~~

$\lim_{x \to 2} f(x) = \frac{2-1}{3} = \frac{1}{3}$

(5) 4. (A) The limit of the product is the product of the limits.

(B) The derivative of the product is the product of the derivatives.

( ) Both are true   (X) A is true and B is false

( ) Both are false   ( ) A is false and B is true.

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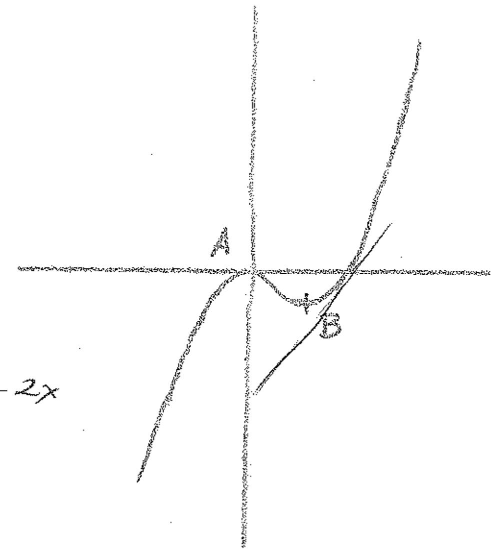
(20) 5. The graph of $y = x^2(x-1)$ at the right:

a. Find the exact coordinates of the relative maximum and minimum points A and B.

b. Find the equation of the tangent line(s) to the curve where it crosses the x axis.



a. $\frac{dy}{dx} = 2x(x-1) + x^2 = 2x^2 - 2x + x^2 = 3x^2 - 2x$

$x(3x-2) \quad x=0,\ x=\frac{2}{3}$

$x = \frac{2}{3} \quad y = \left(\frac{2}{3}\right)^2\left(-\frac{1}{3}\right) = -\frac{4}{27}$

$(0,0) \quad \boxed{\left(\frac{2}{3}, -\frac{4}{9}\right)}$

b. $x = 0,\ x = 1$

$\boxed{y = 0}$

$1(1) = 1$

$\frac{y-0}{x-1} = 1 \quad \boxed{y = x-1}$

(10) 6. Using only the definition, find the derivative of $f(x) = 2x^2-1$.

(For partial credit find $f'(1)$ instead. For extra credit find the derivative of $f(x) = \sqrt{x+1}$ instead).

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 - 1 - (2x^2-1)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2(x^2 + 2x\Delta x + \Delta x^2) - 1 - 2x^2 + 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4x\Delta x + 2\Delta x^2}{\Delta x} = \lim_{\Delta x \to 0} (4x + 2\Delta x) = 4x$$

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3e) 7. Use the formulas to find following derivatives:

a. $f(x) = 5x^6 + x - 6 + \sqrt{x}$

$x^{1/2}$

$6 \cdot 5x^5 + 1 + \frac{1}{2}x^{-1/2} = 30x^5 + 1 + \frac{1}{2\sqrt{x}}$

b. $g(z) = z^{2/3} + \frac{1}{z^2}$ z^{-2}

$g'(z) = \frac{2}{3}z^{-1/3} - 2x^{-3}$

c. $f(x) = (2x^2 - x + 3)^2$

$(2x^2 - x + 3)(4x - 1) + (2x^2 - x + 3)(4x - 1)$

$= 2(4x-1)(2x^2 - x + 3)$

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d. $f(x) = (x^3 - 6x)(2x - 6\sqrt{x})$

$(x^3 - 6x)\left[2 - \frac{6}{2}x^{-1/2}\right] + (2x - 6\sqrt{x})(3x^2 - 6)$

e. $f(x) = \frac{6x}{x^3 - 6}$

$f'(x) = \frac{(x^3 - 6)(6) - 6x(3x^2)}{(x^3 - 6)^2}$

$= \frac{6x^3 - 18x^3 - 36}{(x^3 - 6)^2} = \frac{-12x^3 - 36}{(x^3 - 6)^2}$

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