Part I was last test reworked!

MATH 131
Final Exam
Part II
December 10, 1979

Name KEY

S-U ? ______ Drop one test? ______

Show work!

(5) 1. Let

$$f(x) = \begin{cases} x^2\frac{1}{x^2+1}, & x < -1 \\ \frac{x^2-9}{x-3}, & x > -1, \quad x \neq 3 \end{cases}$$

a. Find $\lim_{x \to -1^-} f(x) = \frac{1}{2}$ $\qquad \lim_{x \to -1^+} f(x) = 2$ $\qquad \frac{1-9}{-1-3} = -2$

b. Is it possible to find values of f(x) for x = -1,3 so that f is continuous? If so what are they?

$x = -1$ no $\quad x = 3$ yes 6

c. Find $\lim_{x \to \infty} f(x) = \infty$ $\qquad \lim_{x \to -\infty} f(x) = 0$

(6) 2. Find the following derivatives:

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a. $\frac{d^2}{dx^2}(x\sqrt{x} + \cos 3x) =$ $\quad \frac{3}{2}x^{1/2} - 3\sin 3x$

$x^{3/2} + \cos 3x$

$\frac{3}{2}\sqrt{x} - 3\sin 3x$

$\boxed{\frac{3}{4}x^{-1/2} - 9\cos x}$

b. $\frac{d}{dx}\sqrt{1 + \sin x} = \frac{1}{2}(1+\sin x)^{-1/2}\cos x$

$\frac{1}{2}(1+\sin x)$ $\quad = \frac{\cos x}{2\sqrt{1+\sin x}}$

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c. $\frac{d}{dx}\sin(x^3-3) = \cos(x^3-3) \cdot 3x^2$

d. $\frac{d}{dx}\left[\frac{x-1}{2x+1}\right] = \frac{(2x+1) - 2(x-1)}{(2x+1)^2} = \frac{3}{(2x+1)^2}$

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5) 7. A projectile is fired up into the air. Its height is given by $h = 800t - 16t^2$ feet from the ground, t seconds after firing.

a. Is it going up or down when t = 10?
b. What is the acceleration when t = 1 ?
c. How high does it go?

$$V = \frac{dh}{dt} = 800 - 32t$$

$$a = -32$$

a. $t = 10$ $V = 800 - 320 > 0$ up

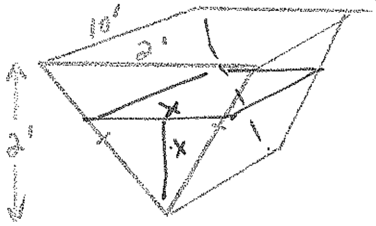
b. -32

c. $800 - 32t = 0$ $\quad h = 800(25) - 16(25)^2$ $\quad \frac{625}{16}$

$\quad\quad\quad\quad\quad\quad\quad\quad 20000 \quad 10,000$

$t = \frac{800}{32} = \frac{100}{4} = 25$ $\quad$ $h = 10,000$

9) 8. A tank with triangular ends is shown below. The ends are 2 feet wide by 2 feet high, and the tank is 10 feet long. (sound familiar?) At what rate is the water level rising when the water is flowing in at the rate of 2 cu.ft. per hour and the water is 1 foot deep?



$$\frac{dV}{dt} = 2 \quad x = 1$$

$$2 = 10 \frac{dx}{dt}$$

$$V = \frac{1}{2}x^2 \cdot 10 = 5x^2 \qquad \frac{dx}{dt} = .2 \text{ ft/hours}$$

$$\frac{dV}{dt} = 10x\frac{dx}{dt}$$

7) 9. A manager is going to order q units of a bulk product (so q need not be integer valued.). If he charges a price of p, he can sell all that is ordered. He estimates that p is a linear function of q. He also estimates that p = 2 when q = 1 and that p = 0 when q = 3. The unit cost is 2 dollars.

a. Write the function for p in terms of q.

$(2,1)$ $\quad (1,2)$ $\quad \frac{p}{q} = -1$ $\quad p = -(q-3)$

$(0,3)$ $\quad (3,0)$ $\quad\quad\quad\quad\quad = -q + 3$

b. Write the revenue function R = pq, in terms of q. What q will make R a maximum?

$$R = (-q+3)q = -q^2 + 3q$$

$$\frac{dR}{dq} = -2q + 3 \quad q = \frac{3}{2}$$

c. What q will maximize total profit?

$$P = R - C = -q^2 + 3q - 2q$$

$$= -q^2 + q$$

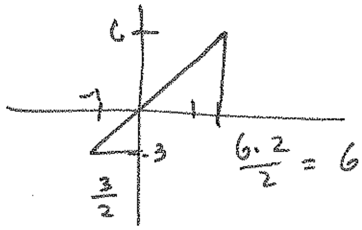
$$\frac{dP}{dq} = -2q + 1 \quad q = \frac{1}{2}$$

2 (cont)

e. $\frac{d}{dx}\left[(x^2-1)\sin x\right] =$ $(x^2-1)\cos x + \sin x(2x)$

(3) 3. What is the anti derivative of $(2x-3)^3$? $\frac{(2x-3)^4}{8}$

(3) 4. Use geometry to evaluate $\int_{-1}^{2} 3x\,dx$. $6 - \frac{3}{2} = 4\frac{1}{2}$



(4) 5. Use Newton's method to evaluate $\sqrt{7}$. Use two iterations with a starting value of 3.

$f(x) = x^2 - 7 = 0$ $f'(x) = 2x$

$f(3) = 9 - 7 = 2$

$f'(3) = 6$

$x_1 = 3 - \frac{2}{6} = 3 - \frac{1}{3} = 2\frac{2}{3} = \frac{8}{3}$

$= 2.6\overline{6}$

$f(\frac{8}{3}) = .11\overline{1}$

$f'(\frac{8}{3}) = \frac{16}{3}$

$x_2 = \frac{8}{3} - .0283$

$= 2.64583$

$\sqrt{7} = 2.64575$

(4) 6. Use the definition of the derivative to find $\frac{dy}{dx}$ for $y = \sqrt{3x-1}$.

$$\frac{\sqrt{3(x+\Delta x)-1} - \sqrt{3x-1}}{\Delta x} \cdot \frac{\sqrt{\ } + \sqrt{\ }}{\sqrt{\ } + \sqrt{\ }} = \frac{3x + 3\Delta x - 1 - (3x-1)}{\Delta x\left(\sqrt{3(x+\Delta x)-1}\ \sqrt{3x-1}\right)}$$

$$= \frac{3\Delta x}{\Delta x\ \sqrt{\ } + \sqrt{\ }} = \frac{3}{\sqrt{3(x+\Delta x)-1} + \sqrt{3x-1}}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{3}{\sqrt{\ } + \sqrt{\ }} = \frac{3}{2\sqrt{3x-1}}$$