MATH 131
TEST II
OCT. 26, 1979

NAME KEY M 42

05

Show work for partial credit.

(8) 1. Let $f(x) = \begin{cases} x^2 & x < 2 \\ 2 & \\ 1, & x \geq 2 \end{cases}$

a. $\lim_{x \to 2^+} f(x) =$ 1

b. $\lim_{x \to -2} f(x) =$ 2

c. $\lim_{x \to 2^-} f(x) =$ 2

d. For what values of x is f not continuous? $x = 2$

(5) 2. Let $f(x) = \dfrac{2x^2}{x^2 + 1}$

a. $\lim_{x \to \infty} f(x) =$ 2

b. $\lim_{x \to -1} f(x) =$ 1

c. For what values of x is f discontinuous? none

(3) 3. Find $\lim_{x \to -1} \dfrac{x + 3}{x^2 - 9}$. $= \lim \dfrac{4}{-8} = -\dfrac{1}{2}$

16

(8) 4. Let $f(x) = \begin{cases} \frac{1}{x+1}, & x < 0 \\ k, & x = 0 \\ x+1, & x > 0 \end{cases}$

1

a. $\lim_{x \to +1} f(x) =$ 2

b. $\lim_{x \to -1^+} f(x) =$ $+\infty$

c. $\lim_{x \to -\infty} f(x) =$ 0

d. Is there a value of k which will make f continuous at x = 0? If so, find it.

$k = 1$

(8) 5 Using the definition, find the following derivatives. (Use formulas in all other problems.)

a. $f'(3)$ where $f(x) = 5x^2 - x + 2$.

$f(3) = 5 \cdot 9 - 3 + 2 = 44$

$f(3+\Delta x) = 5(3+\Delta x)^2 - (3+\Delta x) + 2$

$= 5[9 + 6\Delta x + \Delta x^2] - 3 - \Delta x + 2$

$= 45 + 30\Delta x + 5\Delta x^2 - 1 - \Delta x$

$= 44 + 29\Delta x + 5\Delta x^2$

$\frac{5x^2 - x + 2 - 44}{x - 3}$

or

$\frac{5x^2 - x - 42}{x - 3}$

$\frac{(x-3)(5x+14)}{x-3}$

$\frac{\Delta y}{\Delta x} = \frac{29\Delta x + 5\Delta x^2}{\Delta x} = 29 + 5\Delta x$

$\frac{dy}{dx} = \lim_{\Delta x \to 0} (29 + 5\Delta x)$

$= 29$

$\to 29$

b. $\frac{d}{dx}\left(\frac{3}{2x-1}\right) = \lim_{\Delta x \to 0} \frac{\frac{3}{2(x+\Delta x)-1} - \frac{3}{2x-1}}{\Delta x} = \lim_{\Delta x \to 0} \frac{3}{\Delta x}\left(\frac{1}{2x+2\Delta x-1} - \frac{1}{2x-1}\right)$

$= \lim_{\Delta x \to 0} \frac{3}{\Delta x} \frac{2x-1-(2x+2\Delta x-1)}{(2x+2\Delta x-1)(2x-1)} = \lim_{\Delta x \to 0} \frac{(3) - 2\Delta x}{\Delta x(\ )(\ )}$

$= \lim_{\Delta x \to 0} \frac{-6}{(2x+2\Delta x-1)(2x-1)} = \frac{-6}{(2x-1)^2}$

(12) 6. Find the following derivatives (using the formulas):

a. $\frac{d}{dx}(x^3 - 6x^2 + 3)$ $3x^2 - 12x$

b. $\frac{d}{dx}(2x^{20} + \sqrt{x})$ $40x^{19} + \frac{1}{2}x^{-1/2}$

c. $\frac{d}{dx}(\frac{2}{x} + x^{4/3})$ $-2x^{-2} + \frac{4}{3}x^{1/3}$

d. $\frac{d}{dx}(x\sqrt{x})$ $\frac{3}{2}x^{1/2}$

e. $\frac{d}{dx}(\frac{x^2 + 5}{x})$ $1 - 5x^{-2}$

$x + 5x^{-1}$

(3) 7. The curves $y = 2x^3 - 6$ and $y = x^2 + x - 6$ intersect at $(1,-4)$. Are they perpendicular at that point?

$6x^2$ $2x + 1$

$x = 1$ 6 3 no

(3) 8. An object is chopped from a cliff. Its distance from the ground t seconds after it is dropped is given by the function $h = 180 - 9.8t^2$, where h is measured in meters.

a. What is the velocity after 4 seconds? (Give units!)

b. How high is the cliff?

$\frac{dh}{dt} = -19.6t$

$19.6 \times 4 = 78.4$

a, 78.4 m/sec

b. 180