

*Review original*  
 $\frac{17}{03}$   
 $\frac{100}{100}$   
*100/100*

Show work!

(10) 1. Let  $f(x) = x^3 - 3x^2 + 6$ .

- (a) For what values of  $x$  is  $f(x)$  increasing? decreasing?
- (b) For what values of  $x$  is the graph concave up? concave down?
- (c) Sketch the graph on graph paper.
- (d) Find the absolute maximum and minimum values for  $-1 \leq x \leq 1$ .

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

a.  $3x(x-2)$

$$x = 0, 2$$

dec $0 < x < 2$ inc $x < 0, x > 2$
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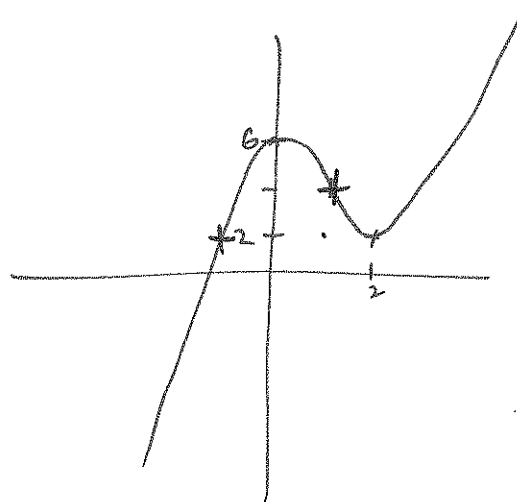
x < 1 cc ↓ x > 1 cc ↑
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d.

$$f(-1) = -1 - 3 + 6 = 2 \text{ MIN}$$

$$f(1) = 1 - 3 + 6 = 4$$

$$f(0) = 6 \text{ MAX}$$

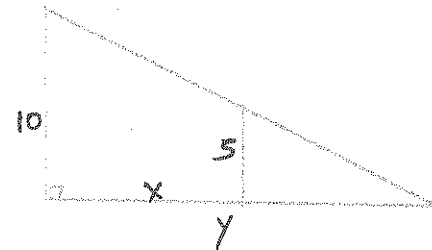


$$8 - 12 + 6 = 2$$

$$1 - 3 + 6 = 4$$

$$-1 - 3 + 6 = 2$$

- (7) 2. A 5 foot man is walking away from a 10 foot light pole at the rate of 3 feet per second. How fast is the tip of his shadow moving when he is 20 feet away?



$$\frac{5}{10} = \frac{x}{x+y}$$

$$5y = 10x - 10x$$

$$-5y = -10x$$

$$y = 2x$$

$$\frac{dy}{dt} = 2 \frac{dx}{dt}$$

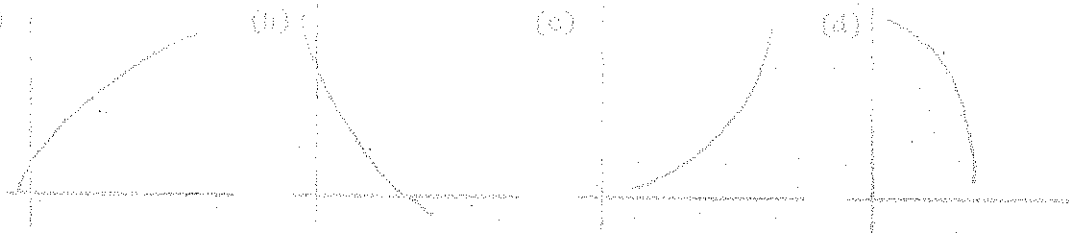
when  $x = 20$

$$\frac{dx}{dt} = 3$$

$$\boxed{\frac{dy}{dt} = 6}$$

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(7) 3. (a) :



Which of the above graphs has  $f'(x) > 0$  and  $f''(x) < 0$ ?

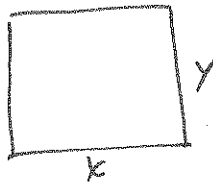
(a)

(7) 4. Find the derivative,  $\frac{dy}{dx}$ , for  $x^3 \cos y + y^2 = 3$ .

$$x^3(-\sin y)y' + 3x^2 \cos y + 2yy' = 0$$

$$y' = \frac{-3x^2 \cos y}{-x^3 \sin y + 2y}$$

(7) 5. A building is being built which is to have one wall made of a more expensive material. The total floor area is to be 10,000 sq. ft. The material for the walls costs \$100 and \$300 per ft. What are the dimensions that will minimize cost?



$$xy = 10,000$$

$$y = 10,000x^{-1}$$

$$C = 100(x + 2y) + 300x$$

$$\begin{aligned} & \frac{dC}{dx} \\ &= 100x + 200(10,000)x^{-2} + 300x \\ &= 400x + 200^6 x^{-1} \end{aligned}$$

$$\frac{dC}{dx} = 400 + 2 \times 10^6 (-x^{-2}) = 0$$

$$x^2 = \frac{2 \cdot 10^6}{400} = \frac{2 \cdot 10^4}{2}$$

$$x = \frac{10^2}{\sqrt{2}} = 50 \cdot 70.7 = 50\sqrt{2}$$

$$y = \frac{10,000}{50\sqrt{2}} = \frac{200}{\sqrt{2}} = 100\sqrt{2}$$

(10) 6.  $y = \frac{3x^2}{(x-2)^2}$

- (a) Sketch the graph of this equation on the attached graph paper. Show work here. Find the coordinates or equations of each of the following and include those on the graph. Use a straight edge.
- (i) Relative maxima and minima
  - (ii) Asymptotes
  - (iii) Points of inflection

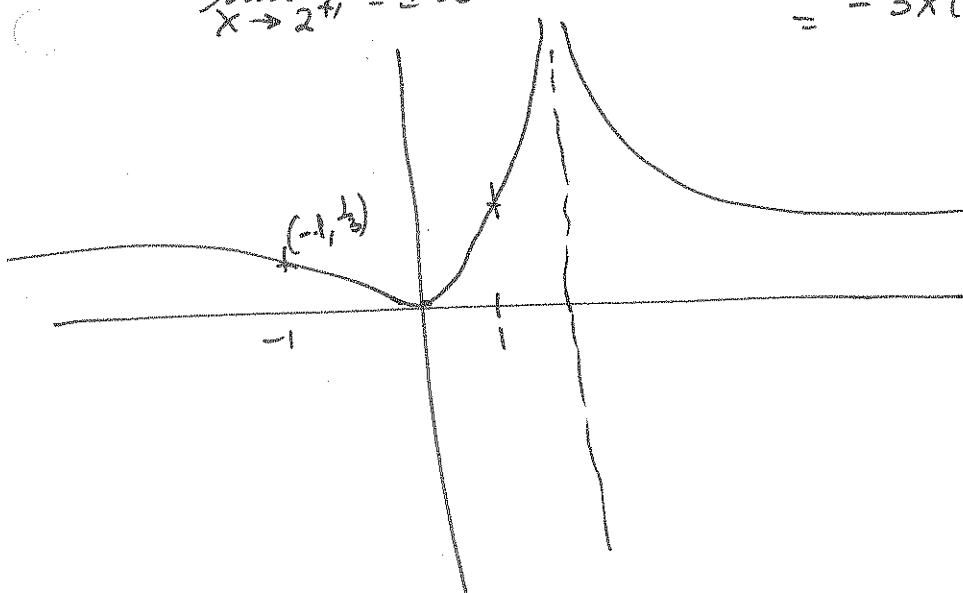
(b) What is the name of this type of function?

$$y' = \frac{(x-2)^2(6x) - 3x^2(2(x-2))}{(x-2)^2} = \frac{6x(x-2)[x-2-x]}{(x-2)^2(6x(x-2))} = \frac{6x(x-2)(-x)}{(x-2)^2(6x(x-2))} = \frac{-x}{(x-2)^2}$$

$\lim_{x \rightarrow \infty} = 3$

$\lim_{x \rightarrow 2^+} = \pm \infty$

$$= \frac{(x-2)3x(x^2-4x)}{(x-2)^2} = \frac{3x(x-2)(-2+x)}{(x-2)^2} = \frac{-3x(x-2)(x+2)}{(x-2)^2} = \frac{6x(x-2)(-x)}{(x-2)^2} = \frac{-12x^2 + 24x}{(x-2)^2}$$



$$\frac{-12x^2 + 24x}{(x-2)^2} = \frac{(x-2)^2(-24x + 24)}{(x-2)^4} = \frac{-(-12x^2 + 24)2(x-2)}{(x-2)^4} = \frac{24(x-2)[(x-2)(-x+1) + 24x^2 - 24x]}{(x-2)^4} = \frac{24(x-2)(-4x-2)}{(x-2)^4}$$

(7) A farmer has 100 tons of grain in storage which can be sold for \$20 a ton. Each week approximately 2 tons will be lost due to spoilage, but the price will go up \$1 a ton. How many weeks should he wait to get the most money?

$x = -1 \quad y = \frac{3}{9} = \frac{1}{3}$

$x = \text{no of weeks.}$

$M = (20+x)(100-2x)$

$= 2000 + 100x - 40x - 2x^2$

$= 2000 + 60x - 2x^2$

$\frac{dM}{dx} = 60 - 4x \quad x = \frac{60}{4} = 15$

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