2:35
43
8 min

MATH 131
TEST III
NOV. 9, 1979

NAME KEY

Show work!

Fuller
Baker

Find the following derivatives: (4 points each)

1. $\frac{d}{dx}[(x^2-1)(x^3+x^2-x-1)]$

$$(x^2-1)(3x^2+2x-1)+(x^3+x^2-x-1)(2x)$$

2. $\frac{d}{dx}\left[\frac{\sqrt{x}}{x^3+3}\right]$ $= \dfrac{(x^3+3)\left(\frac{1}{2}x^{-1/2}\right)-\sqrt{x}\,(3x^2)}{(x^3+3)^2}$

3. $\frac{d}{dx}\cos(3x-1)$ $= -\sin(3x-1)\cdot 3$

4. $\frac{d}{dx}\sin^3(2x)$ $= 3\sin^2 2x\cdot\cos 2x\cdot 2$

5. $\frac{d}{dx}\sqrt{x-\sin x}$ $= \frac{1}{2}(x-\sin x)^{-1/2}(1-\cos x)$

20

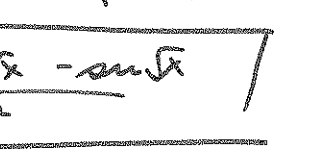
6. $\frac{d^2}{dx^2}(2x-3)^4 = 4(2x-3)^3 \cdot 2 = 8(2x-3)^3$

$\frac{d^2}{dx^2} = 24(2x-3)^2 \cdot 2 = \boxed{48(2x-3)^2}$

7. $\frac{d}{dx} 3 \sin x \cos x \Big|_{x=\frac{\pi}{3}} = 3\cos x(-\sin x) + 3\cos x \cos x \Big|_{x=\frac{\pi}{3}}$

$= 3(\cos^2 x - \sin^2 x)\Big|_{\pi/3}$

$= 3\left[\left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2\right] = 3\left(\frac{1}{4} - \frac{3}{4}\right) = \boxed{-\frac{3}{2}}$



8. $\frac{d}{dx}\frac{\sin\sqrt{x}}{x}$

$= \frac{x\cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}} - \sin\sqrt{x}\cdot 1}{x^2} = \boxed{\frac{\frac{1}{2}\sqrt{x}\cos\sqrt{x} - \sin\sqrt{x}}{x^2}}$

9. $\frac{d}{dx}(1+\sqrt{(x^2-2)})^5$

$5(1+\sqrt{x^2-2})^4\left(\frac{1}{2}(x^2-2)^{-1/2}\cdot 2x\right)$

$5x(1+\sqrt{x^2-2})^4(x^2-2)^{-1/2}$

10. $\frac{d}{dx}[(x^2+x)^2(x-4-\frac{1}{x})^3]$

$(x^2+x)^2\,3(x-4-x^{-1})^2(1+x^{-2}) + (x-4-x^{-1})^3\,2(x^2+x)(2x+1)$

(5) 11. A disc with radius 2 feet is revolving at one rpm. A light is mounted on the edge. The vertical distance of the light from the center is given by $h = 2\sin(2\pi t)$, where t is in minutes.

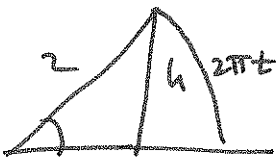
a. Graph the function $h = 2\sin(2\pi t)$ on the last page.

b. What is the vertical velocity when $t = \frac{1}{8}$.

c. Briefly justify the formula for h.

$$\frac{dh}{dt} = 2\cos(2\pi t)\cdot 2\pi$$

$$= 2\cos\left(\frac{\pi}{4}\right)\cdot 2\pi = \frac{4\pi}{\sqrt{2}} = 2\sqrt{2}\pi \text{ ft/min}$$



$$\sin 2\pi t = \frac{h}{2} \qquad h = 2\sin 2\pi t$$

(5) 12. The graphs of $z = g(y)$ and $y = h(x)$ are on the last page. Find

a. z when $x = 3$.

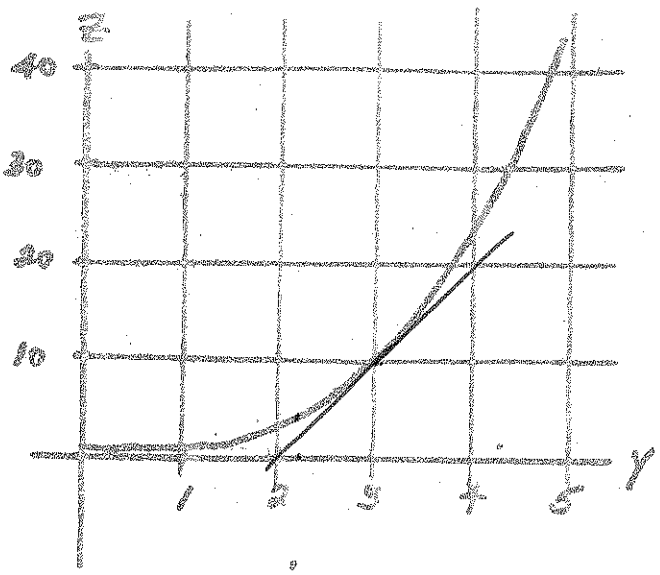
b. $\frac{dz}{dx}$ when $x = 2$.

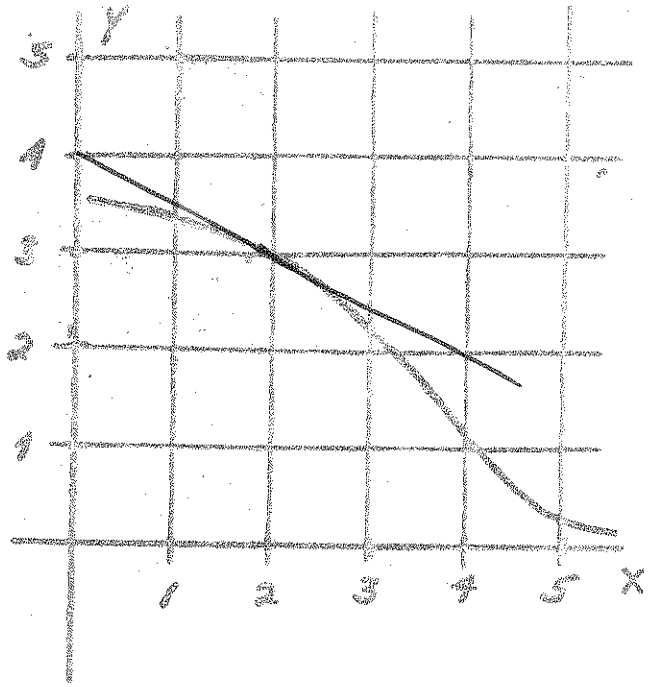
$x = 3 \quad y = 2.2 \quad z = 4$

$x = 2 \quad \frac{dy}{dx} = -\frac{1}{2} \quad y = 3 \quad \frac{dz}{dy} = 10$

$$\frac{dz}{dx} = -\frac{1}{2}\cdot 10 = -5$$

12.





11. $h = 2\sin(2\pi t)$