MATH 131
Review Problems
Final Exam

1. Find the following derivatives:

a. $f'(x)$, where $f(x) = e^{3x^2} \sec x$.

b. $\frac{d^2y}{dx^2}$ where $y = 2 \ln 4x$.

c. $\frac{dy}{dx}$, where $y = \tan(x^2)$

2. Find the following limits ($\pm\infty$ allowed. If one does not exist, say so.)

a. $\lim_{x \to 3} \frac{x-3}{x^2 - 9}$

b. $\lim_{x \to \infty} \frac{2x^2 - 3}{x^2 - 9x}$

c. $\lim_{x \to 0^+} \frac{2}{x^2 - 9x}$

3. Evaluate the following integrals:

a. $\int 4x^3 - \frac{2}{x^2} + e^x \, dx$

b. $\int_0^4 \sqrt{4x + 9} \, dx$

c. $\int \frac{\sin x}{1 + \cos x} \, dx$

d. $\int_0^1 \frac{x}{(x^2 - 4)^3} \, dx$

e. $\int 2 \sin(3x) \, dx$

4. A right triangle has area fixed at 4 in^2. If the base is increasing at the rate of 0.5 in/hr when the base is 4 in., what is the rate of change of the height?

5. Find the equation of the line tangent to the curve $x^2 + xy - 3y^3 = 3$ at the point $(1,-1)$.

6. Sketch the region bounded by the curves $y = x^2 - 3x - 1$, and $y = x - 1$, and find its area.

7. A force of 1000 pounds is required to hold a spring at a extension of 10 feet beyond its natural length. How much work is done when the spring is stretched from this point to 20 feet beyond the natural length?

8. A solid is formed when the region bounded by the curve $y = x^2 + 1$, $x = -1$, $x = 1$, and the x-axis is revolved about the x-axis. Find the volume of this solid. How would you describe its shape?

9. For the function $f(x) = 4x^3$ on the interval $[0,2]$,
   a. Write the Riemann sum with n equal subintervals using x_i^* = right end point. (i.e. the overestimate for area, $\overline{A_n}$)
   b. Find the limit as $n \to \infty$.
   c. What is the proper integral notation for this number?
   d. Check with antiderivatives.

10. A sphere with radius 2 has a hole with radius 1 through its center. Find the volume. (Hint: Think of this as a solid of revolution using $x^2 + y^2 = 4$)

11. For the function $f(x) = x^4 - 6x^2 - 12$
   a. Find coordinates of all local and global maxima and minima.
   b. For what values of x is the function increasing? decreasing?
   c. Find equations of asymptotes, if any.
   d. Find coordinates of points of inflection.
   e. For what values of x is the graph concave up? down?
   f. Carefully sketch the graph, choosing an appropriate viewing window.

12. Find the derivative of each:

   a. $f(x) = (\sqrt{x} + x + 1)(x^2 + 2x + 3)$

   b. $f(x) = \dfrac{x}{x^3 + x + 6}$

   c. $y = \sqrt{x^3 - 8x}$

   d. $y = \dfrac{\sqrt{2x - 3}}{\sqrt{3x^2 - 1}}$

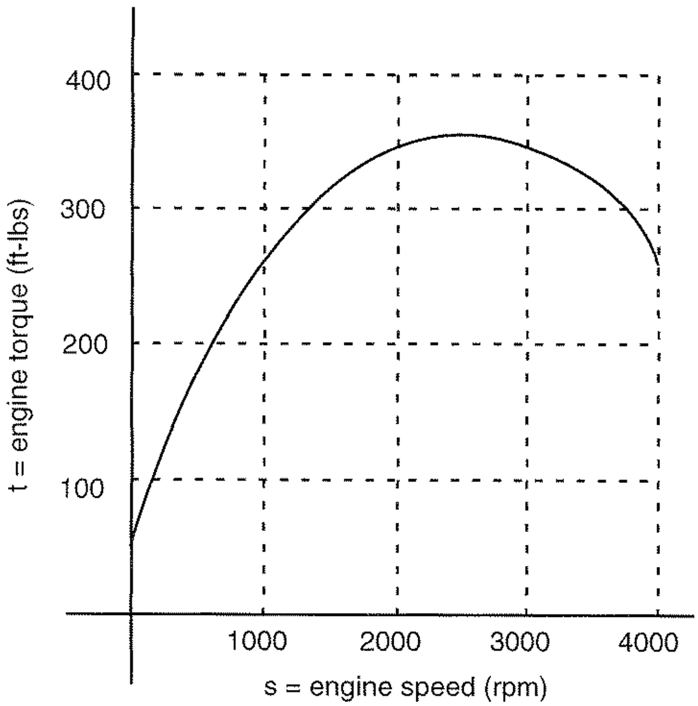
13. Suppose the position of an object after t seconds is $y = -16t^2 + 400t + 1000$ feet above the ground. What is the velocity when t = 10 sec? Is it going up or down at that time? What is the acceleration?

14. Find the equation of the straight line that is tangent to the curve $y = 3x^4 + x$, at the point where x = 1.

15. Using only the <u>definition</u>, find the derivative of: (check with formulas):

$$f(x) = \frac{3}{2x + 1}$$

16. This is the graph of t = torque (twisting force, measured in ft-lbs) as a function of s = engine speed (revolutions per minute) for a gasoline engine.
   a. Estimate the rate of change of torque with respect to engine speed when the speed is 2000 rpm.
   b. At approximately what speed is the torque a maximum?



17. For the function

$$f(x) = \frac{x - 4}{x^2 - 16}$$

   a. For what values of x is the function not continuous?
   b. Can the function be redefined at any of these points to be made continuous? How?

18. For what values of x will the tangent line be horizontal for the curve $y = x^3 - 12x$?

19. Find the following limits (show work, $\pm\infty$ allowed):

a. $\lim_{x\to 2} \frac{\sqrt{x} - 2}{x^2 - 4}$

b. $\lim_{x\to 3} \frac{x + 3}{(x - 3)^2}$

c. $\lim_{x\to 0} \frac{1 - \cos x}{x^2}$

20. Find the following derivatives:

a. $y = e^x \sin x$, find $\frac{dy}{dx}$

b. $f(x) = \sqrt{1 + \tan x}$, find $f'(x)$

c. $y = \cos(2x+1)$, find $\frac{d^2y}{dx^2}$

d. $f(x) = \frac{1}{\sqrt{1 + \sin(\ln x)}}$, find $f'(x)$

e. $x^2 + xy + 3y^3 = 5$, find $\frac{dy}{dx}$

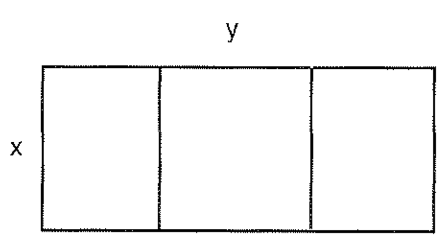
21. Find the equation of the straight line tangent to the curve $x^2 + 4y^3 = -3$ at the point (1,-1).

22. Use Newtons method to find the solution of $x^3 + x + 5 = 0$.

23. What are the global (absolute) maximum and minimum values of $f(x) = x^4 - 2x^2 + 1$ for $0 \le x \le 4$.

24. For what values of x is the function $f(x) = x^3 - 6x + 17$ increasing? decreasing?

25. A rectangular pen is to be built as shown. A total length of fence available is 500 ft. What are the dimensions of the largest pen possible?



26. The formula for the volume of a cone is $V = (1/3)\pi r^2 h$ (with height h and radius of the base r). The height is <u>fixed</u> at h = 3 in. The volume is decreasing at the rate of 1.2 in^3/hr. What is the rate of change of the radius r when r = 2 in.?

27. Graph the function $f(x) = x(x-2)^3$. (Hint: Leave factored.) Give ranges for an appropriate viewing window.

a. Give the x-y coordinates of all local and global maxima and minima.
b. For what x values is the function increasing? decreasing?
c. Give x-y coordinates of points of inflection.
d. For what x values is the graph concave up? concave down?

28. Find the following:

a. $\lim_{x \to 5} \dfrac{x^2 - 25}{x(x-5)}$

b. $\lim_{x \to \infty} \dfrac{x^2 - 25}{2x(x-5)}$

c. $\lim_{x \to 5^+} \dfrac{x^2 - 55}{2x(x-5)}$

d. $y = \dfrac{x - \cos x}{x^2 - 4}$, $\dfrac{dy}{dx} =$

e. $f(x) = \sqrt{x^2 - 3}$, $f''(x) =$

f. $y = \sec(x^2 - 2)$, $\dfrac{dy}{dx} =$

g. $\dfrac{d}{dx}\left(e^{x^2} + \ln x\right)$

29. Find the equation of the tangent line to $3x^2 + 2xy + xy^2 = 6$ at the point (1,1).

30. Find the area of the region bounded by $y = x^3$ and $y = 4x$.

31. Sketch the graph of $f(x) = x^4 + 12x^3$. Choose ranges to display a complete graph. Find exactly (if any):

a. x-y coordinates of relative maxima and minima.
b. x-y coordinates of points of inflection.
c. intervals on which the function is increasing, decreasing.
d. intervals on which the graph is concave up, concave down.

32. A cattle pen is to be built according to the diagram. What are the dimensions of the biggest pen that can be built with 500 feet of fence?

33. A solid is formed by revolving about the x-axis the region bounded by $y = 2x^2$, $y = 2$, and the y-axis. What is the volume?

34. A 6 foot man is walking away from a 20 foot light post at 5 feet per minute. How fast is the length of his shadow changing when he is 10 feet from the post? Does it matter how far he is away?

35. Integrate the following:

a. $\int \sqrt{x} - \sin x \, dx =$

b. $\int_0^{\pi/2} e^{2x} + \cos 2x \, dx$

c. $\int \sqrt{3x - 7} \, dx$

d. $\int_3^4 \frac{x}{x^2 - 4} \, dx$

36. Compute the derivative of

$f(x) = \sqrt{x}$

a. Using formulas:

b. Using only the definition:

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Some answers

1. a. e^{3x^2} sec x tan x + sec x e^{3x^2} 6x, b. $-2x^{-2}$, c. $\sec^2(x^2)$ 2x;
2. a. 1/6, b. 2, c. $-\infty$;
3. a. $x^4 + 2/x + e^x + C$; b. 49/3; c. -ln(1+cos x) + C; d. -7/576; e. -2/3 cos 3x + C
4. -.25 in/hr.;
5. y = 1/8 x - 9/8;
6. 32/3;
7. 15,000 ft-lbs.;
8. $56\pi/15$;
10. $4\pi\sqrt{3}$;
11. a. local max at ((0,12), no global max, local and global mins at $(\pm\sqrt{3},-21)$;
b. I: $-\sqrt{3} < x < 0$, $x > \sqrt{3}$, D: $x < -\sqrt{3}$, $0 < x < \sqrt{3}$; c. none; d. (±1,-17); e. Up x < -1, x > 1, Down -1 < x < 1;
12. a. $(x^{1/2}+x+1)(2x+2)$ + $((x^2+2x+3)(.5x^{-1/2}+1)$; b. $[(x^3+x+6)-x(3x^2+1)]/(x^3+x+6)^2$;
c. 1/2 $(x^3-8x)^{-1/2}(3x^2-8)$; d. $[(3x^2-1)^{1/2}(2x-3)^{-1/2}$ - $3x(2x-3)^{1/2}(3x^2-1)^{-1/2}]/(3x^2-1)$;
13. 80 ft/sec up, -32 ft/sec/sec;
14. y = 13x - 9;
16. a. 0.038 ft-lbs/rpm, b. 2400 rpm.;
17. a. x = ±4, b. x=4, yes, 1/8, x=-4 no;
18. x=±2;
19. a. limit does not exist, but there are one sided limits ($\pm\infty$), redo problem with 4 in place of 2 in numerator: 1/32, b. $+\infty$, c. 1/2;
20. a. $e^x(\cos x + \sin x)$, b. $.5(1+\tan x)^{-1/2}\sec^2 x$. c. -4 cos(2x+1),
d. $-.5(1+\sin(\ln x))^{-3/2}\cos(\ln x)/x$, e. $(-2x-y)/(x+9y^2)$;
21. y=-1/6 x- 5/6;
22. -1.515980228;
23. max 225, min 0;
24. I $x<-\sqrt{2}$, $x>\sqrt{2}$, D $-\sqrt{2}<x<\sqrt{2}$;
25. x= 62.5 ft by y = 125 ft.;
26. $-.3\pi \approx -.95$ in/hr;
27. a. local and global min at (0.5,-1.6875), no maxima; b, I x>.5, D x < .5;
c. (2,0), (1,-1); d. Up x>2, x<1, Down 1<x<2;

E&P