MATH 131
Final Exam
May 10, 1991

Name _____ 158

S-U? ___ Final Exam Double Weight? _____

105/158 = .67%

m = 104

Time OK

$$\sum_{j=1}^{N} j = \frac{1}{2}N^2 + \frac{1}{2}N; \quad \sum_{j=1}^{N} j^2 = \frac{1}{3}N^3 + \frac{1}{2}N^2 + \frac{1}{6}N; \quad \sum_{j=1}^{N} j^3 = \frac{1}{4}N^4 + \frac{1}{2}N^3 + \frac{1}{4}N^2$$

1. Find the derivative of: (20)

a. $x^4 \cos x + \csc 4x$

$x^4(-\sin x) + 4x^3\cos x - \csc 4x \cot 4x \cdot 4$

↑ many forgot

b. $\dfrac{\sin x}{\sqrt{x} - x}$

$$\frac{(\sqrt{x}-x)\cos x - \sin x\left(\frac{1}{2}x^{-1/2}-1\right)}{(\sqrt{x}-x)^2}$$

8/29
12 close

19/29
7 close

c. $\sqrt{1 + \sin(3x-1)}$

$\frac{1}{2}(1+\sin(3x-1))^{-1/2}(\cos(3x-1))\cdot 3$

16

$= \frac{3}{2}\dfrac{\cos(3x-1)}{\sqrt{1+\sin(3x-1)}}$

2. Find: (8)

$\lim_{x \to -3} \dfrac{x^2 - 9}{x + 3} = \lim_{x \to -3} \dfrac{(x+3)(x-3)}{x+3} = \lim_{x \to -3}(x-3)$

$= -3-3 = -6$

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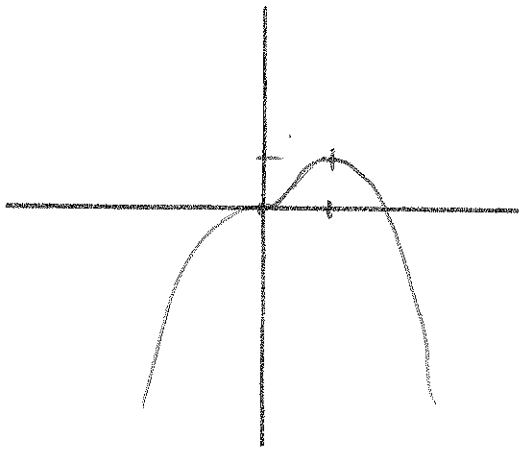
3. Sketch the graph of $f(x) = 4x^3 - 3x^4$, and find (if any): (10)

a. critical points

b. points of inflection



$f'(x) = 12x^2 - 12x^3$

$= 12x^2(1-x)$

$x = 0, 1$

or $(0,0)$, $(1,1)$

6 more

7

13 gut pts not graph

3 gut grad not pts

$f''(x) = 24x - 36x^2$

$= 12x(2-3x)$

PI $x = 0, x = \frac{2}{3}$

$\left(\frac{2}{3}, \frac{16}{27} = .5926\right)$

$12 \cdot \frac{4}{9} - 12\frac{8}{27}$ $\frac{16}{9}$

$12 - x - 12 \cdot \frac{4}{9}$

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4. Find the equation of the straight line which is tangent to the curve $x^3 + y^2 = 3$ at the point (-1,-2). (8) 10

$3x^2 + 2y\frac{dy}{dx} = 0$

$3(-1)^2 + (-4)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{3}{4}$

$y+2 = \frac{3}{4}(x+1) = \frac{3}{4}x + \frac{3}{4}$

$y = \frac{3}{4}x - \frac{5}{4}$

8/29

5. A semi-circular oil spill is spreading from an oil tanker beached on the coast of Kuwait. When the oil slick has spread 10 miles out, it is observed to be spreading at the rate of 2 miles per day. At what rate is the area of the oil spill changing? Extra credit: At what rate is the oil leaking from the tanker? (Suppose that 100 barrels of oil will cover one square mile.) (10) ← reverse

$A = \frac{1}{2}\pi r^2$

$\frac{dA}{dt} = \pi r \frac{dr}{dt}$

$r = 10 \quad \frac{dr}{dt} = 2$

$\frac{dA}{dt} = \pi(10)2$

$= 20\pi$ sq mi/day.

2000π b/d

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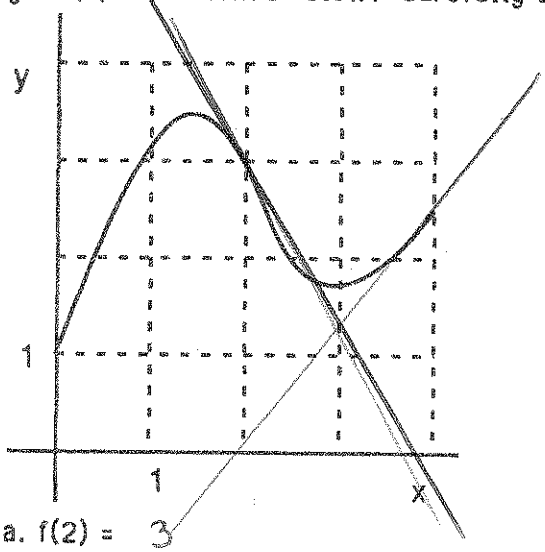
6. y = f(x) is sketched below. Carefully approximate the following: (20)



4 a. f(2) = 3

5 b. f'(2) = (2,3) (3.8,0) $\frac{3}{-1.8} = 1.66$

4 c. The absolute maximum and minimum *values* for f on [0,4]? MAX 3.9 MIN 1.1

4 d. If $g(t) = f(t^2)$:

i. g(2) = f(4) = 2.5

ii. g'(2) = $g f'(t^2) \cdot 2t$

$f'(4) \cdot 4 = 1.25 \cdot 4 = 5$

$f'(4) = (3,0)\ (4, 2.5)$

$\frac{2.5}{2} = 1.25$

NO ONE

40

7. Let the function f be given by (20)

$$f(x) = \begin{cases} 2\,, & x \le 0 \\ \dfrac{x-1}{x^2}\,, & x > 0 \end{cases}$$

a. $\lim_{x \to 0^-} f(x) =$ 2

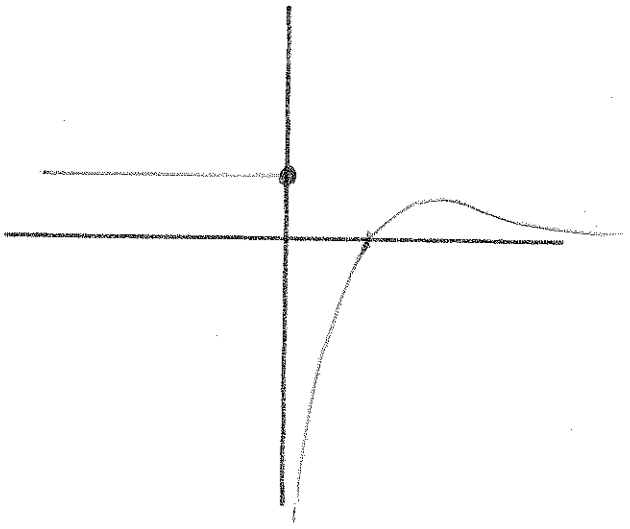
b. $\lim_{x \to 0^+} f(x) = -\infty$

9

c. $\lim_{x \to \infty} f(x) =$ 0

d. Sketch the graph of f. Include zeros, asymptotes, critical points.

3 each
12 close



(1

zero $x = 1$

C.P.

$$f'(x) = \frac{x^2 - (x-1)(2x)}{x^4}$$

$$= \frac{x^2 - 2x^2 + 2x}{x^4}$$

$$= -\frac{x^2 + 2x}{x^4} = x\left(\frac{-x+2}{x^4}\right)$$

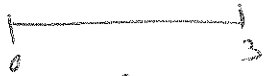
C.P. $x = 2$

$\left(2, \frac{1}{4}\right)$

8. Find the following integral as the limit of Riemann sums, and check. (10)

$$\int_0^3 4x^2\,dx$$

(N=3 partial credit)



$\Delta x = \frac{3}{N}$

$0 \quad \frac{3}{N} \quad 2\cdot\frac{3}{N} \cdots N\cdot\frac{3}{N}$

$C_j = j\frac{3}{N}$

$$\sum_{j=1}^{N} 4\left(j\tfrac{3}{N}\right)^2 \tfrac{3}{N}$$

$$= \sum_{j=1}^{N} \frac{108}{N^3} j^2 = \frac{108}{N^3} \sum_{j=1}^{N} j^2$$

$$= \frac{108}{N^3}\left(\frac{1}{3}N^3 + \frac{1}{2}N^2 + \frac{1}{6}N\right)$$

$$\longrightarrow \frac{108}{3} = (36)$$

8

6 close

$$\checkmark \int_0^3 4x^2\,dx = \frac{4x^3}{3}\Big|_0^3 = \frac{4\cdot 3^3}{3} = 36$$

30

Note $4\cdot 3^2$ also 36 shows correct f

9. Find the following integrals: (20)

a. $\int 4x^3 - \frac{2}{\sqrt{x}} + \sin x \, dx$ $= x^4 - 2\frac{x^{1/2}}{1/2} - \cos x + C$

$4x^3 - 2x^{-1/2} + \sin x$ $= x^4 - 4\sqrt{x} - \cos x + C$

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8 done

18

b. $\int x\sqrt{x^2-3}\, dx = \int \frac{1}{2}\sqrt{u}\, du = \frac{1}{2}\frac{u^{3/2}}{3/2} + C$

$u = x^2 - 3$ $= \frac{1}{3}(x^2-3)^{3/2} + C$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

12
8 done

c. $\int_0^3 2x\sqrt{x+1}\, dx = \int_1^4 2(u-1)\sqrt{u}\, du$

$u = x+1$ $= \int_1^4 2u^{3/2} - 2u^{1/2}\, du$ $= \frac{124}{5} - \frac{36}{3}$ (10)

$du = dx$ $= \frac{2u^{5/2}}{5/2} - \frac{2u^{3/2}}{3/2}\Big|_1^4$ $= \frac{124-50}{5}$ $\frac{232}{15}$

$x = u-1$ $= \boxed{74/5}$

$x=0 \quad u=1$

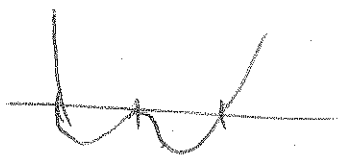
$x=3 \quad u=4$ $= \frac{4}{5}4^{5/2} - \frac{4}{3}4^{3/2} - \left(\frac{4}{5} - \frac{2}{3}\right) = \frac{128}{5} - \frac{32}{3} - \frac{4}{5} + \frac{2}{3}$

10. Find the area between the curve $y = x^4 - x^2$ and the x-axis. (10)

$x^4 - x^2 = 0$ ± 2 next time between curves

$x^2(x^2-1) = 0$

$x = 0, \; x = \pm 1$



$-\int_{-1}^{1} x^4 - x^2\, dx = -2\int_0^1 x^4 - x^2\, dx$ (15)

$= -\left(\frac{x^5}{5} - \frac{x^3}{3}\right)\Big|_{-1}^{1}$

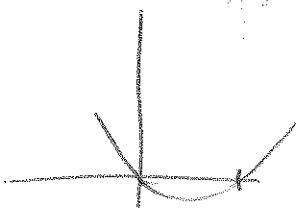
$= -2\left(\frac{x^5}{5} - \frac{x^3}{3}\right)\Big|_0^1 = -2\left(\frac{1}{5} - \frac{1}{3}\right)$

$= -2\left(\frac{-2}{15}\right)$

$= \frac{4}{15}$

30

11. Find the volume of the solid generated by revolving about the x-axis the region bounded by $y = x^2 - x$ and $y = 0$. (10)



$x^2 - x \geq 0$

$x(x-1)$

$$\int_0^1 \pi (x^2-x)^2 \, dx$$

9

6 dud y axis

$$= \pi \int_0^1 x^4 - 2x^3 + x^2 \, dx$$

$$= \pi \left(\frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3} \right) = \pi \left(\frac{6-15+10}{30} \right) = \frac{\pi}{30}$$

12. Using only the DEFINITION, find $f'(x)$ for $f(x) = \sqrt{x}$. (10)
[Or for partial credit, find $f'(4)$.]

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$$\lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$= \lim_{\Delta x \to 0} \frac{x+\Delta x - x}{\Delta x \left(\sqrt{x+\Delta x} + \sqrt{x} \right)} = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x \left(\sqrt{x+\Delta x} + \sqrt{x} \right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

13. Extra credit: What is the volume of a sphere of radius 4 which has a hole of radius one drilled through it? (10)

know

I quit 5

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