allowed 28 min

MATH 131
Quiz VII
March 21,1991

Name Key

mean 20.5/30 (70%)
med 24

10

1. The surface area of a sphere is given by $A = 4\pi r^2$, where r is the radius. If the radius is decreasing at the rate of 2 in/hour when the radius is 10 inches, how fast is the surface area changing?

8 got all

$A = 4\pi r^2 \qquad \frac{dr}{dt} = -2$ when $r = 10$

watch sign units

many missed sign & units

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi(10)(-2)$

$= -160\pi$ sq in/hr.

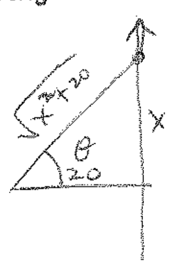
dec.

only one couldn't do it

θ

2. An airplane is traveling north in a straight line and is picked up by a radar station which is located 20 miles west of the path. The beam locks onto the plane. When the angle that the beam makes with due east is $\pi/4$, the radar is turning at a rate of 9 rad/hr. At that instant how fast is the plane moving?

7 full



$\frac{x}{20} = \tan\theta$

$\frac{1}{20}\frac{dx}{dt} = \sec^2\theta \frac{d\theta}{dt}$

$\sec\frac{\pi}{4} = \sqrt{2}$

$\frac{dx}{dt} = 20(\sec\frac{\pi}{4})^2(9)$

$\frac{d\theta}{dt} = 9$ rad/hr when $\theta = \frac{\pi}{4}$

$= 20(2)\cdot 9$

$= 360$ mi/hr

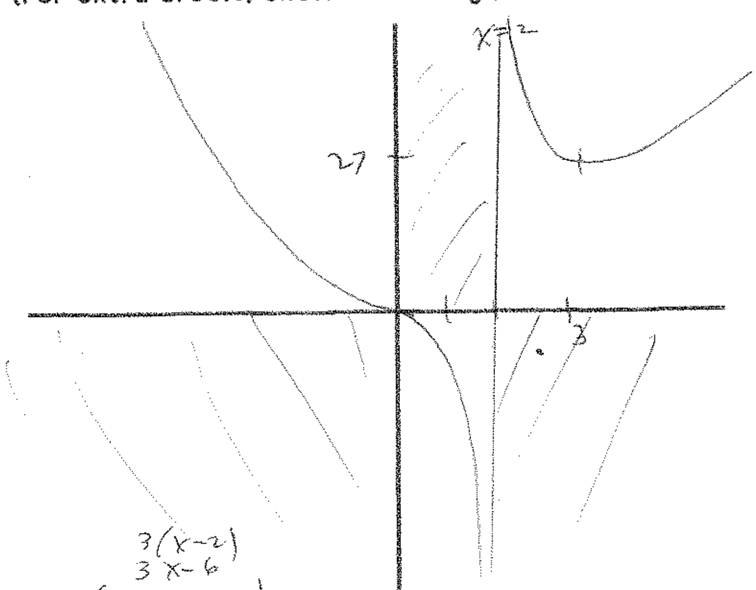
1 got all
3 close

~~12~~ 10

3. Carefully sketch the graph of
$y = \frac{x^3}{x-2}$
Find zeros, asymptotes, and critical points, if any.
(For extra credit, check concavity.)

2 full

2 got all many close



zero $x=0$

VA $x = 2$

HA $\frac{x^3}{x-2} \approx \frac{x^3}{x} = x^2 \to \infty$

none

$\frac{dy}{dt} = \frac{(x-2)(3x^2) - \cancel{(x-2)}x^3}{(x-2)^2}$

$= \frac{x^2[3(x-2) - x]}{(x-2)^2}$

$= \frac{x^2(2x-6)}{(x-2)^2}$

CP. $x = 0,\ x = 3$

$(0,0) \quad (3,27)$

PI

$3(x-2)$
$3x-6$
$2x(2x-6+x)$

$\frac{(x-2)^2[2x(2x-6) + x^2(2)] - x^2(2x-6)\,2(x-2)}{(x-2)^4}$

$\frac{2x(x-2)[3(x-2)(x-2)] - x(2x-6)]}{(x-2)^4}$

$3x^2-12x+12$

$\frac{2x[3(x^2-4x+4) - 2x^2 + 6x)}{(x-2)^3} = \frac{2x(x^2-6x+12)}{(x-2)^3}$

$x = 6 \pm \sqrt{36-48}$

$x=0$
PI